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Gaussian Anti Magic Labeling on Joining of Eiffel Tower Graph, E – Graph, Fire Cracker Graph and Fork Graph

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Abstract: Gaussian antimagic labeling which is defined as $G(p,q)$. A function $f: V \rightarrow \{\, g + ih \,/\, g,h \in N\}, 1 \leq g < h \leq q$, so that we interpret the produced function $\tau: E \rightarrow N$ described by $f^*(ab) = |f(a)|^2 + |f(b)|^2$. Solution of all the elements are well defined and different. A graph accepts Gaussian antimagic labeling is known as Gaussian antimagic graph. This study we demonstrate the persistence of Gaussian antimagic labeling for some graphs like Joining of Eiffel Tower graph, E – graph, Fire Cracker graph and Fork graph.

Keywords: Graph Labeling, Gaussain antimagic, Joining of Eiffel Tower graph, E – graph, Fire Cracker graph and Fork graph.

I. Preface

Graph labelling is a process of assigning of integers to the vertices or edges or both subject to certain conditions. A valuable survey to know about the many graph labeling methods is done by J.A. Gallian recently [1,2,3,4]. The join of two or more graphs is an operation that connects every vertex of one graph to every vertex of the other graph. Gaussian antimagic labeling used in many graphs by Thirusangu & Selvaganapathy [5,6,7,8,9]. This study we demonstrate the persistence of Gaussian antimagic labeling for some special graphs like joining of Eiffel Tower graph, E – graph, Fire Cracker graph and Fork graph.

II. Preliminaries

Within this part we deliver fundamental definitions and additional details those are essential condition for the current study.

Definition: 2.1 The Eiffel Tower graph is a plane graph tree with seven edges and seven vertices.

Definition: 2.2 The graph that is connected if the corresponding edges in the original graph are adjacent is called as E – graph.

Definition: 2.3 The Fire Cracker graph γ(k,l) is the graph acquired by the series of kl – stars by connecting one leaf from each .

Definition: 2.4 The Fork graph is tree with five vertices and four edges.

III. Primary Solutions

Theorem 3.1: A graph which verifies Gaussian antimagic labeling is the joining of Eiffel Tower.

Proof: Entitle $V = \{v_1, v_2, v_3, \ldots, v_{7r}\}$ be the apex and the joining of edges of the Eiffel Tower graph.

$E = \{\{ v_{7q-6}\ v_{7q-4} / 1 \le q \le r, r \in N \} \cup \{ v_{7q-5}\ v_{7q-4} / 1 \le q \le r, r \in N \} \cup \{ v_{7q-4}\ v_{7q-3} / 1 \le q \le r, r \in N \} \cup \{ v_{7q-4}\ v_{7q-2} / 1 \le q \le r, r \in N \} \cup \{ v_{7q-3}\ v_{7q-2} / 1 \le q \le r, r \in N \} \cup \{ v_{7q-3}\ v_{7q-1} / 1 \le q \le r, r \in N \} \cup \{ v_{7q-2}\ v_{7q} / 1 \le q \le r, r \in N \} \cup \{ v_{7q-6}\ v_{7q+1} / 1 \le q \le r, r \in N \}\}$

Function can be defined as $f: V \to \{g + ih /\ g, h \in N,\ h = g + 1,\ 1 \le g \le n\}$ so that $f(v_t) = t + i(t+1), 1 \le t \le n.$

To interpret the produced function $\tau: E \to N$ so that $f^*(ab) = |f(a)|^2 + |f(b)|^2$.

To acquire the labels assigned to edges:

$f^*(v_{7q-6}\ v_{7q-4}) = 196q^2 - 280q + 102,\ 1 \le q \le r,\ r \in N$

$f^*(v_{7q-5}\ v_{7q-4}) = 196q^2 - 224q + 66,\ 1 \le q \le r,\ r \in N$

$f^*(v_{7q-4}\ v_{7q-3}) = 196q^2 - 168q + 38,\ 1 \le q \le r,\ r \in N$

$f^*(v_{7q-4}\ v_{7q-2}) = 196q^2 - 140q + 30,\ 1 \le q \le r,\ r \in N$

$f^*(v_{7q-3}\ v_{7q-2}) = 196q^2 - 112q + 18,\ 1 \le q \le r,\ r \in N$

$f^*(v_{7q-3}\ v_{7q-1}) = 196q^2 - 84q + 14,\ 1 \le q \le r,\ r \in N$

$f^*(v_{7q-2}\ v_{7q}) = 196q^2 - 28q + 6,\ 1 \le q \le r,\ r \in N$

Thus $f^*(E) = \{18, 326, \ldots\ldots, 196r^2 - 280r + 102; 38, 402, \ldots\ldots, 196r^2 - 224r + 66; 66, 486, \ldots\ldots, 196r^2 - 168r + 38; 86, 534, \ldots\ldots, 196r^2 - 140r + 30; 102, 578, \ldots\ldots, 196r^2 - 112r + 18; 126, 630, \ldots\ldots, 196r^2 - 84r + 14; 174, 734, \ldots\ldots, 196r^2 - 28r + 6; 150, 626, \ldots\ldots, 196r^2 - 112r + 66\}$ all the elements are well defined and different.

However, the graph which verifies Gaussian antimagic labeling is the joining of Eiffel Tower.

Theorem 3.2: A graph which verifies Gaussian antimagic labeling is the joining of E.

Proof: Entitle V = {v_1, v_2, v_3, …, v_{6r}} be the apex and the joining of edges of the E – graph.

E ={{ v_{6q-5} v_{6q-4} / $1 \le q \le r$, $r \in N$ } ∪ { v_{6q-3} v_{6q-2} / $1 \le q \le r$, $r \in N$ } ∪ { v_{6q-1} v_{6q} / $1 \le q \le r$, $r \in N$ } ∪ { v_{6q-5} v_{6q-3} / $1 \le q \le r$, $r \in N$ }∪ { v_{6q-3} v_{6q-1} / $1 \le q \le r$, $r \in N$ } ∪ { v_{6q} v_{6q+1} / $1 \le q \le r$, $r \in N$ }}

Function can be defined as $f: V \to \{g + ih /\ g, h \in N,\ h = g + 1,\ 1 \le g \le n\}$ so that $f(v_t) = t + i(t + 1), 1 \le t \le n$.

To interpret the produced function $\tau: E \to N$ so that $f^*(ab) = |f(a)|^2 + |f(b)|^2$.

To acquire the labels assigned to edges:

$f^*(v_{6q-5}, v_{6q-4}) = 144q^2 - 192q + 66$, $1 \le q \le r$, $r \in N$
$f^*(v_{6q-3}, v_{6q-2}) = 144q^2 - 96q + 18$, $1 \le q \le r$, $r \in N$
$f^*(v_{6q-1}, v_{6q}) = 144q^2 + 2$, $1 \le q \le r$, $r \in N$
$f^*(v_{6q-5}, v_{6q-3}) = 144q^2 - 168q + 54$, $1 \le q \le r$, $r \in N$
$f^*(v_{6q-3}, v_{6q-1}) = 144q^2 - 72q + 14$, $1 \le q \le r$, $r \in N$
$f^*(v_{6q}, v_{6q+1}) = 144q^2 + 48q + 6$, $1 \le q \le r$, $r \in N$

Thus f *(E) = {18, 258,……,$144r^2 - 192r + 66$; 66, 402,……,$144r^2 - 96r + 18$; 146, 578,……,$144r^2 + 2$; 30, 294,……,$144r^2 - 168r + 54$; 86, 446,……,$144r^2 - 72r + 14$; 198, 678,……,$144r^2 + 48r + 6$} all the elements are well defined and different.

However, the graph which verifies Gaussian antimagic labeling is the joining of E.

Theorem 3.3: A graph which verifies Gaussian antimagic labeling is the joining of Fire Cracker.

Proof: Entitle V = {v_1, v_2, v_3, …, $v_{(n+1)r}$} be the apex and the joining of edges of the Fire Cracker graph

E ={{v_q v_{q+1} / $1 \le q \le r - 1$, $r \in N$} ∪ { v_q v_{q+1} / $r+1 \le q \le 2r - 1$, $r \in N$} ∪ { v_q v_{q+1} / $2r+1 \le q \le 3r - 1$, $r \in N$} ∪ { v_q v_{q+1} / $3r+1 \le q \le 4r - 1$, $r \in N$} ∪ { v_q v_{q+1} / $(n-1)r+1 \le q \le nr - 1$, $r \in N$} ∪ { v_q v_{q+1} / $nr+1 \le k \le (n+1)r - 1$, $r \in N$}}

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Function can be defined as $f: V \rightarrow \{g + ih \,/\, g, h \in N,\ h = g + 1,\ 1 \leq g \leq n\}$ so that $f(v_t) = t + i(t + 1), 1 \leq t \leq n$.

To interpret the produced function $\tau: E \rightarrow N$ so that $f^*(ab) = |f(a)|^2 + |f(b)|^2$.

To acquire the labels assigned to edges:

$f^*(v_q, v_{q+1}) = 4q^2 + 8q + 6,\ 1 \leq q \leq r-1,\ r \in N$
$f^*(v_q, v_{q+1}) = 4q^2 + 8q + 6,\ r+1 \leq q \leq 2r-1,\ r \in N$
$f^*(v_q, v_{q+1}) = 4q^2 + 8q + 6,\ 2r+1 \leq q \leq 3r-1,\ r \in N$
$f^*(v_q, v_{q+1}) = 4q^2 + 8q + 6,\ 3r+1 \leq q \leq 4r-1,\ r \in N$
$f^*(v_q, v_{q+1}) = 4q^2 + 8q + 6,\ (n-1)r+1 \leq q \leq nr-1,\ r \in N$
$f^*(v_q, v_{q+1}) = 4q^2 + 8q + 6,\ nr+1 \leq q \leq (n+1)r-1,\ r \in N$

Thus $f^*(E) = \{(18, 38, 66, \ldots\ldots, 4r^2 - 8r + 6, 4r^2 + 2), (4r^2 + 16r + 18, 4r^2 + 24r + 38, \ldots\ldots, 16r^2 - 16r + 6, 16r^2 + 2), (16r^2 + 32r + 18, 16r^2 + 48r + 38, \ldots\ldots, 36r^2 + 2), (36r^2 + 48r + 18, 36r^2 + 72r + 38, \ldots\ldots, 64r^2 + 2), (4n^2 r^2 + 4r^2 - 8nr^2 + 16rn - 16r + 18, 4n^2 r^2 + 4r^2 - 8nr^2 + 24rn - 24r + 38, \ldots\ldots, 4n^2 r^2 - 8rn + 6, 4n^2 r^2 + 2), (4n^2 r^2 + 16rn + 18, 4n^2 r^2 + 24rn + 38, \ldots\ldots, 4n^2 r^2 + 4r^2 - 8r + 6, 4n^2 r^2 + 4r^2 + 8nr^2 + 2)\}$ all the elements are well defined and different.

However, the graph which verifies Gaussian antimagic labeling is the joining of Fire Cracker.

Theorem 3.4: A graph which verifies Gaussian antimagic labeling is the joining of Fork.

Proof: Entitle $V = \{v_1, v_2, v_3, \ldots, v_{5r}\}$ be the apex and

$E = \{\{ v_{5q-4} v_{5q-3} \,/\, 1 \leq q \leq r, r \in N \} \cup \{ v_{5q-3} v_{5q-2} \,/\, 1 \leq q \leq r, r \in N \} \cup \{ v_{5q-3} v_{5q-1} \,/\, 1 \leq q \leq r, r \in N \} \cup \{ v_{5q-1} v_{5q} \,/\, 1 \leq q \leq r, r \in N \} \cup \{ v_{5q} v_{5q+1} \,/\, 1 \leq q \leq r, r \in N \}\}$

the joining of edges of the Fork graph.

Function can be defined as $f: V \rightarrow \{g + ih \,/\, g, h \in N,\ h = g + 1,\ 1 \leq g \leq n\}$ so that $f(v_t) = t + i(t + 1), 1 \leq t \leq n$.

To interpret the produced function $\tau: E \rightarrow N$ so that $f^*(ab) = |f(a)|^2 + |f(b)|^2$.

To acquire the labels assigned to edges:

$f^*(v_{5q-4}, v_{5q-3}) = 100q^2 - 120q + 38,\ 1 \leq q \leq r,\ r \in N$

$f^*(v_{5q-3}, v_{5q-2}) = 100q^2 - 80q + 18,\ 1 \leq q \leq r,\ r \in N$

$f^*(v_{5q-3}, v_{5q-1}) = 100q^2 - 60q + 14,\ 1 \leq q \leq r,\ r \in N$

$f^*(v_{5q-1}, v_{5q}) = 100q^2 + 2,\ 1 \leq q \leq r,\ r \in N$

$f^*(v_{5q}, v_{5q+1}) = 100q^2 + 10q + 6,\ 1 \leq q \leq r,\ r \in N$

Thus $f^*(E) = \{18, 198, \ldots\ldots, 100r^2 - 120r + 38; 38, 258, \ldots\ldots, 100r^2 - 80r + 18; 54, 294, \ldots\ldots, 100r^2 - 60r + 14; 102, 402, \ldots\ldots, 100r^2 + 2; 146, 486, \ldots\ldots, 100r^2 + 40r + 6\}$ all the elements are well defined and different.

However, The graph which verifies Gaussian antimagic labeling is the joining of Fork.

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