

An Explainable, Privacy-Preserving AI Framework for Credit Risk in Nepalese Cooperative Banks: A Federated Survival Graph Bayesian Approach

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Abstract

Nepal's cooperative banking sector is in serious trouble. By mid-July 2025, non-performing loans across the country's banking and financial institutions had climbed to Rs 258.19 billion, and the cooperative sector accounts for a large share of this problem. Forty cooperatives alone embezzled Rs 87.89 billion from over 60,000 depositors, many of them farmers, housewives, retirees, and daily wage laborers. The National Cooperative Bank Limited reported a capital adequacy ratio of just 0.82 percent, which points to technical insolvency rather than mere distress. The International Monetary Fund has warned that further cooperative failures could threaten the stability of Nepal's entire banking system. The paper proposes a new theoretical architecture for credit risk management in Nepalese cooperative banks, which we call the Federated Survival Graph Bayesian Framework (FSGBF). Conventional binary classification approaches predict whether a borrower will default, but they cannot tell us when. Our framework addresses this

through Cox proportional hazards survival analysis. It captures network contagion effects using Graph Neural Networks that map borrower interdependencies within cooperative communities. Cross-institutional learning is enabled through Federated Learning with Differential Privacy, so that 31,450 primary cooperatives can collectively strengthen their models without sharing sensitive member data. Epistemic uncertainty is quantified through Bayesian Neural Networks with Monte Carlo Dropout, which helps distinguish between data noise and genuine model ignorance. Causal explanations come through interventional Shapley values, which identify which features correlate with default and which features actually cause changes in default risk when manipulated. The paper proves the consistency of the Cox partial likelihood estimator, derives the Breslow baseline hazard estimator, and establishes the convergence of GraphSAGE under neighborhood sampling. We also prove the FedAvg convergence theorem under convexity and bounded gradient assumptions, derive the Rényi differential privacy composition rule, and show that Monte Carlo Dropout approximates variational inference with a Bernoulli variational posterior. The four Shapley axioms for the interventional (causal) Shapley value are verified, and the bias-variance decomposition for the composite FSGBF predictor is established. Drawing on Nepal Rastra Bank's Financial Stability Report FY 2024/25, the National Cooperative Federation of Nepal's 2025 sectoral snapshot, parliamentary investigation findings, and the IMF's July 2024 country report, we situate this mathematical framework within Nepal's institutional reality. The sector has 7.4 million members, 56 percent of them women, holding Rs 847 billion in deposits. Yet it operates under split oversight, where 25,000 of 32,000 cooperatives are financial intermediaries with minimal real-time supervision. The paper argues that without predictive, network-aware, privacy-preserving, uncertainty-quantified, and causally explainable credit risk management, Nepal's cooperative movement will collapse under the weight of its own opacity, and millions of livelihoods will go with it.

Keywords: Survival Analysis, Graph Neural Networks, Federated Learning, Differential Privacy, Bayesian Neural Networks, Cooperative Banks, Nepal, Credit Risk, Cox Proportional Hazards, Causal Shapley Values, Rényi Differential Privacy, Monte Carlo Dropout.

1. Introduction

there's a particular cruelty to financial fraud when it wears the mask of community trust. In Nepal, the cooperative movement was constitutionally enshrined as one of the three pillars of national economic development, alongside the public and private sectors. precisely because it promised to democratize finance for those whom commercial banks ignored. The Constitution of Nepal (2015) recognized cooperatives as vehicles for poverty reduction, gender empowerment, and rural development. The numbers, on paper, still look impressive: 31,450 primary cooperatives, 7.4 million members comprising 23 percent of the national population, Rs 847 billion in deposits, Rs 453 billion disbursed as loans, 94,002 employees, and 56 percent female membership with 40 percent women on cooperative boards. These aren't marginal institutions. they're the financial system for nearly a quarter of Nepal. But the paper conceals the rot. The Nepal Rastra Bank's

Financial Stability Report for FY 2024/25 records that non-performing loans across all banking and financial institutions rose to Rs 258.19 billion by mid-July 2025, up from Rs 199.66 billion the previous year, pushing the system-wide NPL ratio to 4.62 percent. Within the cooperative sector specifically, the National Cooperative Bank Limited, the apex institution of the movement, reported a non-performing loan ratio of 33.01 percent and a capital adequacy ratio that collapsed to 0.82 percent. Its core capital to risk-weighted assets ratio fell to 0.34 percent. These are not numbers that indicate a sector experiencing difficulties. These are numbers that indicate a sector experiencing existential crisis. And then there is the fraud. The parliamentary special committee chaired by Surya Thapa, formed in May 2024, documented that 40 cooperatives had embezzled Rs 87.89 billion from depositors. The Swarnalakshmi Multipurpose Cooperative alone allegedly diverted over Rs 1.19 billion from more than 5,000 depositors through unauthorized checks and fictitious loans. The Suryadarshan Cooperative in Pokhara siphoned Rs 1.12 billion. The Supreme Cooperative in Butwal transferred over Rs 150.31 million. Over 600 individuals have been arrested. The chairman of Gorkha Media Network, GB Rai, remains a fugitive with an Interpol diffusion notice. Former Home Minister and Rastriya Swatantra Party chair Rabi Lamichhane has been implicated in check fraud exceeding Rs 800 million. The victims aren't wealthy speculators. They are low-income retirees, small farmers, housewives, and daily-wage laborers who deposited modest sums expecting community-based safety. Over 60,000 of them have seen their life savings evaporate. Some have descended into despair or suicide. The International Monetary Fund, in its July 2024 country report, did not mince words: it warned that more failures of cooperative lenders could endanger the soundness of Nepal's banking system, attributing the crisis to "investment in illiquid and underperforming assets, impacting repayment capacity. combined with some cases of fraud," and explicitly recommended establishing a specialized regulatory authority to collect data, regulate, supervise, and re-license financial cooperatives. Thirteen suggestion committees have been formed since 2024. Reports have been submitted. Little has changed. What makes this crisis hard to justify is that the tools to prevent it already exist. They are not hidden in classified laboratories or locked behind multinational patents. they're published in open academic literature, implemented in freely available software libraries, and validated across financial systems from Scandinavia to sub-Saharan Africa. The problem isn't availability but that Nepal's cooperative sector has never been equipped with a credit risk architecture that's simultaneously predictive (forecasting default before it occurs), relational (accounting for the social and economic networks that bind borrowers together), privacy-preserving (enabling collective learning without collective exposure), uncertainty-aware (admitting what the model doesn't know), and causally explainable (distinguishing correlation from actionable intervention). This work builds that architecture. We reject the binary classification model that dominates credit risk literature, the simplistic "will default / will not default" framing that tells a loan officer nothing about when a borrower might fail, how network effects might cascade through a village savings group, or whether rejecting a loan based on a correlated feature like geographic location is statistically justified or merely

discriminatory. Instead, we build the Federated Survival Graph Bayesian Framework (FSGBF), a five-layer theoretical system that integrates:

1. *Survival Analysis via Cox Proportional Hazards*, modeling the time-to-default as a continuous stochastic process rather than a binary outcome, enabling dynamic loan pricing, early warning systems, and provisioning schedules tied to expected remaining lifetime.
2. *Graph Neural Networks (GraphSAGE and GAT)*, representing borrowers as nodes in a dynamic graph where edges encode kinship, village proximity, supply-chain relationships, and co-guarantor obligations, capturing the contagion dynamics that cause cooperative defaults to cluster geographically and socially.
3. *Federated Learning with Differential Privacy*, allowing Nepal's 31,450 primary cooperatives to train a collective global model by sharing only encrypted parameter updates, rarely raw member data, a important capability in a sector where data centralization would create catastrophic privacy risks and regulatory violations.
4. *Bayesian Neural Networks with Monte Carlo Dropout*, replacing point-estimate predictions with full posterior predictive distributions, distinguishing aleatoric uncertainty (inherent borrower volatility) from epistemic uncertainty (model ignorance due to data scarcity), and flagging predictions that require human adjudication because the model admits it does not know.
5. *Causal Shapley Values*, extending standard SHAP explanations from associative to interventional reasoning, enabling loan officers to answer counterfactual questions: "If I helped this borrower diversify her income sources, how much would her default hazard decrease?" rather than merely "Which features are associated with default?"

This is not an empirical paper. We don't claim to have deployed this framework in Nepalese cooperatives and achieved specific accuracy metrics, because the data infrastructure, labeled survival records, and network graphs required for such validation do not yet exist at scale. What we offer is the complete mathematical blueprint, derived from first principles, grounded in Nepal's verified institutional data, and aligned with the Cooperative Act 2017, the Nepal Rastra Bank's Fourth Strategic Plan (2022–2026), and the IMF's reform recommendations. The mathematics is the argument. The urgency is the context. The framework is the only honest response to a sector that is mathematically insolvent, socially betrayed, and institutionally exposed. We make eight contributions. First off, we construct the first mathematical framework that simultaneously addresses temporal prediction (survival analysis), relational structure (graph neural networks), privacy (federated learning with differential privacy), uncertainty (Bayesian neural networks), and causal explainability (interventional Shapley values) for credit risk in cooperative banking. Also, we prove the consistency of the Cox partial likelihood estimator under standard regularity conditions, providing the statistical foundation for survival-based credit scoring. Third, we derive the GraphSAGE convergence theorem, establishing that the node embeddings converge to a population-limit embedding as the graph size and neighborhood sample size grow. Fourth, we

prove the FedAvg convergence theorem under convexity and bounded gradient assumptions, with explicit convergence rate $\mathcal{O}(1/\sqrt{T})$ for non-convex objectives. Fifth, we derive the Rényi differential privacy composition rule and the moments accountant for the Gaussian mechanism, providing tight privacy budget tracking across federated rounds. Sixth, we prove that Monte Carlo Dropout approximates variational inference with a Bernoulli variational posterior, and derive the aleatoric-epistemic uncertainty decomposition. Seventh, we verify the four Shapley axioms (efficiency, symmetry, dummy, additivity) for the interventional (causal) Shapley value and prove its consistency under causal sufficiency. Eighth, we establish the bias-variance decomposition for the composite FSGBF predictor and derive Rademacher complexity generalization bounds for the federated GNN-Cox model. The remainder of this paper is organized as follows. Section 2 reviews the related literature. Section 3 develops the theoretical framework and mathematical foundations: survival analysis (§3.1), graph neural networks (§3.2), federated learning with differential privacy (§3.3), Bayesian neural networks with Monte Carlo Dropout (§3.4), and causal Shapley values (§3.5), each with full proofs of the key theorems. Section 4 specifies the FSGBF methodology: system architecture (§4.1), the survival graph model (§4.2), federated training with differential privacy (§4.3), Bayesian uncertainty quantification at inference (§4.4), and causal explainability (§4.5). Section 5 establishes the convergence and generalization theory: FedAvg convergence, GraphSAGE convergence, privacy-utility tradeoff, and Rademacher bounds. Section 6 discusses policy implications and regulatory alignment. Section 7 addresses challenges, limitations, and ethical considerations. Section 8 concludes. Four appendices collect the notation table, the full proof of Cox partial likelihood consistency, the Rényi DP composition proof, and the MC Dropout variational inference proof.

2. Literature Review

2.1 From Binary Classification to Survival Analysis in Credit Risk

The dominant framework in machine learning for credit risk treats default as a binary classification problem: given a borrower's features, predict whether default will occur within a fixed horizon, typically 12 or 24 months. This framing, while computationally convenient, discards important temporal information. A borrower who defaults in month 3 is not equivalent to one who defaults in month 23; the former indicates immediate distress requiring intervention, while the latter may reflect gradual deterioration amenable to restructuring. Binary classification can't distinguish these cases. Survival analysis, originating in biostatistics with Kaplan and Meier (1958) and Cox (1972), models the time until an event occurs as a continuous random variable. In credit risk, the event is default, and the survival function $S(t) = P(T > t)$ gives the probability that a loan survives beyond time t . The hazard function $h(t) = \lim_{\Delta t \rightarrow 0} P(t \leq T < t + \Delta t \mid T \geq t) / \Delta t$ gives the instantaneous default rate conditional on survival to time t . These functions enable dynamic risk pricing, lifetime value estimation, and provisioning that evolves with loan seasoning, capabilities that binary classifiers fundamentally lack. Recent literature has begun integrating survival analysis with machine learning. DeepSurv (Katzman et al., 2018) employed neural

networks within the Cox proportional hazards framework, demonstrating superior concordance index compared to traditional Cox regression on high-dimensional data. Random survival forests (Ishwaran et al., 2008) extended ensemble methods to right-censored survival data. However, these approaches remain underexplored in cooperative banking contexts, where small sample sizes, high censoring rates (most loans don't default within the observation window), and heterogeneous borrower populations create unique statistical challenges. The theoretical foundations we develop in Section 3.1, including the full proof of partial likelihood consistency (Theorem 3.1). address these challenges directly.

2.2 Network Effects and Graph Neural Networks in Credit Risk

Traditional credit risk models assume borrower independence, the default probability of one farmer in Surkhet is assumed uncorrelated with that of her neighbor. This assumption is economically absurd. In Nepalese cooperatives, borrowers are linked by kinship, village proximity, shared agricultural supply chains, joint liability groups, and co-guarantor arrangements. A drought affects entire villages simultaneously. A livestock disease outbreak cascades through cooperative members who purchased animals from the same supplier. The default of a prominent local entrepreneur triggers confidence crises among her creditors and employees. Graph Neural Networks (GNNs) explicitly model these relational dependencies. GraphSAGE (Hamilton et al., 2017) generates node embeddings by sampling and aggregating features from local neighborhoods, enabling inductive learning on unseen nodes. Graph Attention Networks (GAT) (Veličković et al., 2018) extend this with learned attention weights that quantify the importance of each neighbor. In credit risk, GNNs have demonstrated notable predictive gains by exploiting borrower similarity structures. Wang et al. (2025) proposed IG-DP, a graph neural network-based default prediction method for imbalanced financial datasets, using unlabeled background nodes to enhance prediction performance on the DGraphFin dataset with over 3.7 million nodes and 4.3 million dynamic edges. Risk.net (2026) evaluated GraphSAGE and GAT for credit default prediction, finding that optimized GNNs consistently outperform logistic regression, random forest, and gradient boosting in AUC and F1 metrics while maintaining resilience under adversarial feature perturbations. Zhang et al. (2025) combined conditional tabular GANs with GraphSAGE for inductive credit risk learning, achieving 7.78 percent accuracy improvement and 13.35 percent AUC-ROC increase over non-augmented baselines. For Nepalese cooperatives, where social networks are dense and default contagion is real, GNNs aren't merely an algorithmic refinement; they're a structural necessity.

2.3 Federated Learning and Privacy-Preserving Finance

The cooperative sector's fragmentation, 31,450 primary institutions, many with only hundreds of members. means that no single cooperative possesses sufficient data to train a solid credit risk model. Yet centralized data pooling is politically impossible (cooperatives are autonomous member-owned institutions), legally fraught (Nepal lacks full data protection legislation but the constitutional right to privacy is increasingly invoked), and practically dangerous (centralizing

financial data creates a single point of failure for fraud). Federated Learning (FL), introduced by McMahan et al. (2017), offers an alternative. In FL, each institution trains a local model on its private data and shares only model parameter updates with a central server, which aggregates them into a global model. The raw data never leaves the local institution. For credit risk, this enables collective model improvement without collective data exposure. However, FL alone is insufficient. Model updates can leak sensitive information through membership inference and model inversion attacks. Differential Privacy (DP), formalized by Dwork and Roth (2014), provides strict mathematical guarantees: a randomized mechanism $\mathcal{M}$ is (ϵ, δ) -differentially private if the probability of any output changes by at most a factor of e^ϵ when any single individual's data is added or removed. In federated credit risk, DP is implemented by clipping gradient updates and adding Gaussian noise before transmission, ensuring that no individual borrower's data can be reconstructed from the aggregated model. Nature (2026) evaluated privacy-preserving federated credit risk models combining DP and Homomorphic Encryption (HE), finding that standard FL achieves the highest accuracy but lacks protection against inference attacks, while HE-FL preserves performance under encrypted aggregation with negligible model drift, and DP-FL with $\epsilon = 8.65$ yields the most balanced operational outcome. UDSpace (2026) demonstrated theoretically lossless, privacy-preserving federated learning for credit risk prediction with alternative data. For Nepal's cooperative sector, where trust has been shattered by embezzlement scandals, federated learning with differential privacy is not a technical luxury; it is a trust-rebuilding mechanism.

2.4 Bayesian Deep Learning and Uncertainty Quantification

Standard neural networks produce point estimates, a single probability of default for each borrower. In high-stakes lending decisions, point estimates are dangerously misleading. They convey false certainty, masking situations where the model has encountered a borrower profile unlike anything in its training data. A Nepalese cooperative lending to a transgender entrepreneur in a remote mountain village, or a returning migrant with irregular remittance patterns, may receive a confident prediction that is actually extrapolation into model ignorance. Bayesian Neural Networks (BNNs) address this by placing probability distributions over network weights rather than learning fixed values. The posterior predictive distribution $p(y | x, \mathcal{D}) = \int p(y | x, w)p(w | \mathcal{D}) dw$ integrates over all plausible weight configurations, yielding a prediction but a full uncertainty profile. Gal and Ghahramani (2016) proved that Monte Carlo Dropout, applying dropout at test time and averaging over multiple forward passes, approximates Bayesian inference with minimal computational overhead, making BNNs practical for production deployment. Umavezi (2025) demonstrated the application of Bayesian Deep Learning to financial stress testing and risk forecasting, showing that BDL outperforms conventional models in both accuracy and stability by capturing both epistemic (model) and aleatoric (data) uncertainties. In credit risk, this translates to the ability to flag "unknown unknowns", borrower profiles for which the model's epistemic uncertainty is so high that automated decision-making should be deferred to human

judgment. Recent literature demonstrates increasing interest in integrating analytical methods, numerical computation, machine learning, and uncertainty quantification to support complex decision-making. Conventional mathematical models provide theoretical clarity and interpretability, but they may struggle to represent nonlinear, stochastic, and data-intensive systems. Machine-learning models offer predictive flexibility, although their lack of transparency and sensitivity to data quality can limit their use in high-risk management environments. Hybrid frameworks attempt to overcome these limitations by combining mathematical rigor with data-driven adaptability. Sahani, Lee, Pandey, Pandey, and Mandal (2026) provide a broad methodological foundation for hybrid analytical, numerical, and machine-learning approaches to deterministic and stochastic differential equations. Their framework emphasizes stability, convergence, and uncertainty quantification, which are essential when mathematical solutions are applied to real-world systems. Analytical methods help explain the structural behavior of a system, while numerical techniques approximate solutions where closed-form expressions are unavailable. Machine learning can further improve estimation and forecasting by identifying complex patterns from data. The study therefore positions hybridization as a means of balancing theoretical reliability, computational efficiency, and predictive performance. The treatment of uncertainty becomes more advanced in the machine learning–augmented risk-management framework developed by Sahani, Lee, Pandey, Pandey, Jha, and Karna (2026). The authors integrate fractional stochasticity, Wasserstein robustness, and rough-path neural filtering to represent deep uncertainty in Nepalese management systems. Fractional stochastic models are particularly relevant when management processes display memory, persistence, or long-range dependence. Wasserstein robustness can protect decisions against differences between observed and future probability distributions, while neural filtering supports the estimation of complex and partially observable system states. This framework broadens conventional risk management by recognizing that uncertainty may arise not only from random variation but also from incomplete information, model misspecification, and changing environmental conditions. Explainability is another important dimension of intelligent risk management. Sahani, Lee, Pandey, Pandey, Jha, and Karna (2026) examine explainable machine learning in relation to credit-risk management and lending decisions in Nepalese cooperative banks. Their review is significant because cooperative banks often operate with limited datasets, informal borrower information, and locally specific lending practices. Although machine-learning models can improve the identification of default risk, their use in lending requires understandable explanations for borrowers, managers, auditors, and regulators. Explainable machine learning can therefore enhance accountability and managerial confidence by showing which factors influence a credit-risk prediction. Nevertheless, explainability must be considered alongside data privacy, fairness, model validation, and the possibility of historical bias in lending records. The practical value of hybrid modelling is also evident in the healthcare-management framework proposed by Sahani, Lee, Pandey, Pandey, Sah, and Jha (2026). The study connects analytical, numerical, and machine-learning techniques with resource allocation,

operational efficiency, risk forecasting, and strategic decision-making in Nepal. Healthcare systems must allocate limited staff, beds, medicines, equipment, and financial resources under uncertain demand conditions. Hybrid models can support this process by combining optimization with predictive analytics. However, successful implementation depends on the availability of reliable health data, adequate digital infrastructure, trained personnel, and institutional capacity to interpret model outputs. Thus, technical sophistication alone cannot guarantee better healthcare decisions unless it is accompanied by organizational readiness and effective governance. A more specialized application is presented by Sahani, Lee, Pandey, Pandey, Sah, Jha, and Sah (2026), who apply hybrid analytical–numerical–machine-learning architectures to dental practice management. Their framework links mathematical modelling with operational stability, risk management, resource planning, and uncertainty quantification. Dental practices face uncertainties relating to patient demand, appointment cancellations, treatment duration, material requirements, staffing, and equipment availability. Predictive models may improve scheduling and resource utilization, while numerical and analytical methods can provide structured support for operational decisions. This study illustrates how general mathematical and machine-learning concepts can be adapted to a specific service-management context. Sahani, Lee, Pandey, Pandey, Sah, Mandal, and Jha (2026) extend this interdisciplinary approach by incorporating biotechnological catalysts into hybrid analytical–numerical–machine-learning architectures. The study connects biotechnology, mathematical modelling, and risk-aware management within Nepalese systems. Its interdisciplinary orientation suggests that biological and technological processes can be represented through mathematical models and assessed through machine-learning-supported decision systems. However, the integration of heterogeneous biological, operational, and managerial data may create challenges involving measurement consistency, computational complexity, model interpretation, and external validation.

2.5 Causal Explainability: Beyond Associative SHAP

Standard SHAP values, grounded in Shapley's (1953) cooperative game theory, measure associative feature importance: how much does a feature contribute to the prediction, holding the model constant? But association is not causation. A SHAP value may reveal that "living in Province 2" increases default probability, but this may reflect underlying poverty rather than a causal effect of geography. If a cooperative uses this insight to deny loans to all Province 2 applicants, it commits statistical discrimination. Causal Shapley values, developed by Heskes et al. (2020) and extended by Chen et al. (2023), distinguish between features that are causal parents of default (intervening on them changes the outcome) and features that are merely correlated (intervening on them does nothing). By modeling the data-generating process as a structural causal model (SCM) and computing Shapley values under interventions rather than observations, causal XAI enables loan officers to identify actionable levers: "Helping this borrower obtain crop insurance" (causal intervention) rather than "Rejecting this borrower because she is female" (associative correlation). In Nepal's cooperative sector, where gender, caste, and geography are

deeply correlated with economic outcomes, causal explainability is an ethical necessary. Without it, explainable AI becomes merely a more sophisticated tool for perpetuating historical injustice. The formal verification of the four Shapley axioms for the interventional Shapley value (Theorem 3.6 in Section 3.5) ensures that causal explanations retain the same fairness guarantees as their associative counterparts. Recent studies have demonstrated increasing interest in integrating mathematical modelling, machine learning, uncertainty quantification, and risk-aware decision-making within Nepalese management systems. Sahani et al. (2026) proposed a machine-learning-augmented framework combining fractional stochasticity, Wasserstein robustness, and rough-path neural filtering to support risk management under deep uncertainty. Their study emphasized that conventional point predictions are insufficient in volatile management environments and that uncertainty should be explicitly incorporated into model-based decisions. Similarly, Sahani et al. (2026) developed hybrid analytical, numerical, and machine-learning methods for deterministic and stochastic differential equations, establishing theoretical results concerning stability, convergence, and uncertainty propagation. These mathematical foundations are relevant to credit-risk modelling because borrower conditions and institutional risk develop dynamically rather than remaining static. Extending this research direction, Sahani et al. (2026) integrated analytical, numerical, and machine-learning approaches for decision-making, operational stability, and uncertainty analysis in Nepalese management systems. Their framework illustrated how mathematical optimization and predictive learning can complement managerial judgment when institutional information is incomplete or unstable. Comparable hybrid architectures have also been applied to specialized management settings. Sahani et al. (2026d) formulated a mathematical framework for risk management and uncertainty quantification in dental-practice management, while Sahani et al. (2026) examined the incorporation of biotechnological catalysts into hybrid analytical–numerical–machine-learning architectures. Although these studies address different application domains, they collectively demonstrate the flexibility of hybrid computational systems for representing nonlinear relationships, stochastic disturbances, and decision uncertainty. Within Nepal’s health-management context, Sahani et al. (2026) proposed a hybrid framework for resource allocation, operational efficiency, risk forecasting, and strategic decision-making. The study reinforced the importance of combining predictive algorithms with optimization and uncertainty analysis when resources are limited and decisions have meaningful social consequences. More directly related to the present research, Sahani et al. (2026g) developed an explainable machine-learning framework for credit-risk management and intelligent lending decisions in Nepalese cooperative banks. Their mathematical review examined gradient-boosting models, particularly XGBoost and LightGBM, together with SHAP and LIME explanations, information-theoretic feature selection, class-imbalance correction, fairness constraints, convergence analysis, and generalization bounds. This work established an important foundation for transparent machine-learning-based credit assessment in Nepal’s cooperative sector. Nevertheless, the existing frameworks primarily emphasize classification, general risk forecasting,

or domain-specific hybrid decision systems. They do not jointly model the timing of borrower default, relational contagion among interconnected borrowers, privacy-preserving collaboration among cooperatives, epistemic uncertainty, and causal explanations of predicted risk. The present study addresses this gap through the Federated Survival Graph Bayesian Framework. It integrates Cox proportional-hazards survival analysis, graph neural networks, federated learning with differential privacy, Bayesian uncertainty quantification, and interventional Shapley values. Consequently, the proposed framework extends earlier explainable and hybrid machine-learning research from static prediction toward a temporal, network-aware, privacy-preserving, uncertainty-sensitive, and causally interpretable credit-risk architecture for Nepalese cooperative banks.

3. Theoretical Framework and Mathematical Foundations

This section develops the complete mathematical architecture underlying the FSGBF. We begin with survival analysis and the Cox proportional hazards model, proving the consistency of the partial likelihood estimator. We then develop the graph neural network framework, proving the GraphSAGE convergence theorem. We show the federated learning framework with differential privacy, proving the FedAvg convergence theorem and the Rényi DP composition rule. We construct Bayesian neural networks with Monte Carlo Dropout, proving its equivalence to variational inference. We conclude with causal Shapley values, verifying the four Shapley axioms for the interventional value.

3.1 Survival Analysis: The Cox Proportional Hazards Model

Let $T \in \mathbb{R}^+$ denote the continuous random variable representing time-to-default, and let $X \in \mathbb{R}^d$ denote a vector of borrower covariates. The survival function and hazard function are:

$$S(t | X) = P(T > t | X), \quad h(t | X) = \frac{f(t|X)}{S(t|X)} \quad (3.1)$$

where $f(t | X)$ is the probability density function of T given X . The survival and hazard functions are related by:

$$S(t | X) = \exp\left(-\int_0^t h(u | X) du\right) = \exp(-H(t | X)) \quad (3.2)$$

where $H(t | X) = \int_0^t h(u | X) du$ is the cumulative hazard function. The Cox proportional hazards model (Cox, 1972) decomposes the hazard into a baseline hazard $h_0(t)$ shared by all borrowers and a covariate-dependent multiplicative effect:

$$h(t | X) = h_0(t) \exp(\beta^T X) \quad (3.3)$$

The key assumption is proportional hazards: the ratio of hazards for any two borrowers is constant over time:

$$\frac{h(t|X_j)}{h(t|X_i)} = \exp\left(\beta^T (X_j - X_i)\right) \quad (3.4)$$

This semi-parametric formulation avoids specifying the baseline hazard $h_0(t)$, which is left entirely non-parametric. The regression coefficients β are estimated by maximizing the partial likelihood:

$$L(\beta) = \prod_{i:\delta_i=1} \frac{\exp(\beta^T X_i)}{\sum_{j \in R(t_i)} \exp(\beta^T X_j)} \quad (3.5)$$

where $\delta_i = 1$ indicates that borrower i experienced default (uncensored), $\delta_i = 0$ indicates right-censoring (loan still performing or prepaid at observation end), and $R(t_i)$ is the risk set at time t_i , the set of all borrowers still under observation and not yet defaulted just prior to t_i . The log-partial likelihood is:

$$\ell(\beta) = \sum_{i:\delta_i=1} [\beta^T X_i - \log \sum_{j \in R(t_i)} \exp(\beta^T X_j)] \quad (3.6)$$

Maximization proceeds via Newton-Raphson or gradient ascent. The score function (gradient of ℓ) and Hessian are:

$$U(\beta) = \frac{\partial \ell}{\partial \beta} = \sum_{i:\delta_i=1} \left[X_i - \frac{\sum_{j \in R(t_i)} X_j \exp(\beta^T X_j)}{\sum_{j \in R(t_i)} \exp(\beta^T X_j)} \right] \quad (3.7)$$

$$I(\beta) = -\frac{\partial^2 \ell}{\partial \beta \partial \beta^T} = \sum_{i:\delta_i=1} \left[\frac{\sum_{j \in R(t_i)} X_j X_j^T \exp(\beta^T X_j)}{\sum_{j \in R(t_i)} \exp(\beta^T X_j)} - \bar{X}_i(\beta) \bar{X}_i(\beta)^T \right] \quad (3.8)$$

where $\bar{X}_i(\beta) = \frac{\sum_{j \in R(t_i)} X_j \exp(\beta^T X_j)}{\sum_{j \in R(t_i)} \exp(\beta^T X_j)}$ is the weighted average of covariates in the risk set. The

observed information matrix $I(\beta)$ provides standard errors for inference via $SE(\hat{\beta}) = I(\hat{\beta})^{-1/2}$.

Theorem 3.1 (Consistency of the Cox Partial Likelihood Estimator). *Let (T_i, C_i, X_i) , $i = 1, \dots, n$, be i.i.d. triplets where T_i is the default time, C_i is the censoring time, and X_i is the covariate vector. Let $\tilde{T}_i = \min(T_i, C_i)$ be the observed time and $\delta_i = \mathbf{1}\{T_i \leq C_i\}$ the event indicator. Assume:*

- (i) *The censoring mechanism is independent (non-informative): $T \perp C \mid X$.*
- (ii) *The true hazard follows the Cox model: $h(t \mid X) = h_0(t) \exp(\beta_0^T X)$ for some true β_0 .*
- (iii) *The covariate distribution has finite second moment: $\mathbb{E}[\|X\|^2] < \infty$.*
- (iv) *The information matrix $I(\beta_0)$ is positive definite.*

Then the partial likelihood estimator $\hat{\beta}_n = \arg \max_{\beta} \ell_n(\beta)$ is consistent: $\hat{\beta}_n \xrightarrow{p} \beta_0$ as $n \rightarrow \infty$. Also, $\hat{\beta}_n$ is asymptotically normal:

$$\sqrt{n}(\hat{\beta}_n - \beta_0) \xrightarrow{d} \mathcal{N}(0, \Sigma) \quad (3.9)$$

where $\Sigma = I(\beta_0)^{-1} V I(\beta_0)^{-1}$ with $V = \text{Var}(U_i(\beta_0))$ the variance of the per-subject score contribution.

Proof. The proof proceeds in four steps.

Step 1 (Uniform convergence of the normalized log-partial likelihood). Define the normalized log-partial likelihood $\ell_n(\beta)/n$. By the law of large numbers applied to the counting process formulation (Andersen and Gill, 1982), $\ell_n(\beta)/n$ converges uniformly in probability to a deterministic limit function $\ell^*(\beta)$:

$$\frac{\ell_n(\beta)}{n} \xrightarrow{p} \ell^*(\beta) = \mathbb{E}[\delta(\beta^T X - \log \mathbb{E}[\exp(\beta^T X) \mathbf{1}\{\tilde{T} \geq T\} \mid T, \delta = 1])] \quad (3.10)$$

The uniform convergence follows from the Glivenko-Cantelli theorem applied to the class of functions $\{\exp(\beta^T X) \mathbf{1}\{\tilde{T} \geq t\} : \beta \in \mathbb{R}^d, t \geq 0\}$, which is VC-subgraph (and hence Glivenko-Cantelli) under the finite-second-moment assumption (iii).

Step 2 (Identification: β_0 uniquely maximizes ℓ^*). At $\beta = \beta_0$, the score function vanishes: $\nabla \ell^*(\beta_0) = 0$ (this follows from the martingale property of the Cox score process under the true model). The Hessian at β_0 is $-\Sigma^{-1}$ (the negative of the information), which is negative definite by assumption (iv). Therefore β_0 is a strict local maximum of ℓ^* . By concavity of ℓ^* (which follows from the convexity of $\log \sum_j \exp(\beta^T X_j)$), β_0 is the unique global maximum.

Step 3 (Consistency). By the argmax theorem (van der Vaart, 1998, Theorem 5.7): if $\ell_n/n \xrightarrow{p} \ell^*$ uniformly and ℓ^* has a unique maximizer β_0 , then $\hat{\beta}_n \xrightarrow{p} \beta_0$.

Step 4 (Asymptotic normality). By a Taylor expansion of the score function around β_0 :

$$0 = U_n(\hat{\beta}_n) = U_n(\beta_0) - I_n(\beta_n^*)(\hat{\beta}_n - \beta_0) \quad (3.11)$$

where β_n^* is between $\hat{\beta}_n$ and β_0 . By consistency, $\beta_n^* \xrightarrow{p} \beta_0$, and by the law of large numbers, $I_n(\beta_n^*)/n \xrightarrow{p} I(\beta_0)$. By the martingale central limit theorem (Rebolledo, 1980) applied to the counting process, $U_n(\beta_0)/\sqrt{n} \xrightarrow{d} \mathcal{N}(0, V)$ where V is the variance of the per-subject score. Solving for $\hat{\beta}_n$:

$$\sqrt{n}(\hat{\beta}_n - \beta_0) = I_n(\beta_n^*)^{-1} \frac{U_n(\beta_0)}{\sqrt{n}} \xrightarrow{d} I(\beta_0)^{-1} \mathcal{N}(0, V) = \mathcal{N}(0, I(\beta_0)^{-1} V I(\beta_0)^{-1}) \quad (3.12)$$

Under the correctly specified Cox model, $V = I(\beta_0)$ (the information equality), so $\Sigma = I(\beta_0)^{-1}$, recovering the standard sandwich formula. ▀

Proposition 3.1 (Breslow Baseline Hazard Estimator). *The Breslow estimator of the baseline cumulative hazard is:*

$$\hat{H}_0(t) = \sum_{t_i \leq t} \frac{1}{\sum_{j \in R(t_i)} \exp(\hat{\beta}^T X_j)} \quad (3.13)$$

and the baseline survival function is $\hat{S}_0(t) = \exp(-\hat{H}_0(t))$. The predicted survival probability for a borrower with covariates X is:

$$\hat{S}(t | X) = [\hat{S}_0(t)]^{\exp(\hat{\beta}^T X)} \quad (3.14)$$

Proof. The baseline cumulative hazard is estimated by maximizing the full likelihood about H_0 at the estimated $\hat{\beta}$. The full log-likelihood decomposes as $\ell(\beta, H_0) = \ell_P(\beta) + \ell_0(H_0 | \beta)$ where ℓ_P is the partial likelihood and ℓ_0 is the baseline term. Setting $\partial \ell_0 / \partial H_0(t_i) = 0$ at each observed default time t_i yields the Breslow estimator (3.13). The predicted survival (3.14) follows from $S(t | X) = S_0(t) \exp(\beta^T X)$ under the proportional hazards model. ▀

For time-dependent covariates, important in cooperative lending where borrower circumstances change with agricultural seasons, remittance flows, and macroeconomic shocks. the model generalizes to $h(t | X(t)) = h_0(t) \exp(\beta^T X(t))$ and the partial likelihood becomes:

$$L(\beta) = \prod_{i:\delta_i=1} \frac{\exp(\beta^T X_i(t_i))}{\sum_{j \in R(t_i)} \exp(\beta^T X_j(t_i))} \quad (3.15)$$

This formulation enables dynamic risk scoring: a borrower's hazard updates as her savings balance, debt burden, and repayment history develop. In the FSGBF, the Cox model serves as the foundational survival layer, but its linear predictor $\beta^T X$ is replaced by the output of a Graph Neural Network, enabling nonlinear, relational hazard modeling.

3.2 Graph Neural Networks: Message Passing on Borrower Networks

Cooperative borrowers don't exist in isolation. We represent the borrower population as a graph $\mathcal{G} = (V, E, X)$ where V is the set of N borrowers (nodes), $E \subseteq V \times V$ is the set of edges encoding relationships, and $X \in \mathbb{R}^{N \times d}$ is the feature matrix. Edges are constructed from multiple relational signals: - *Kinship edges*: family members, spouses, cohabitants. - *Geographic edges*: borrowers within the same ward or village municipality. - *Economic edges*: members of the same agricultural cooperative, joint liability group participants, supply-chain buyers and sellers. - *Financial edges*: co-guarantors on loans, shared collateral pledges. The adjacency matrix $A \in \{0,1\}^{N \times N}$ encodes these edges, with $A_{ij} = 1$ if borrowers i and j are connected. Graph Neural Networks operate through message passing. At layer k , each node aggregates information from its neighbors $\mathcal{N}(i)$:

$$h_i^{(k)} = \sigma \left(W^{(k)} \cdot AGG \left(\{h_j^{(k-1)} : \forall j \in \mathcal{N}(i)\} \right) \right) \quad (3.16)$$

where $h_i^{(0)} = x_i$ is the initial feature vector, $W^{(k)}$ is a learnable weight matrix, AGG is an aggregation function (mean, max, or LSTM), and σ is a nonlinearity. GraphSAGE (Hamilton et al., 2017) employs neighborhood sampling and concatenation:

$$h_i^{(k)} = \sigma \left(W^{(k)} \cdot CONCAT \left(h_i^{(k-1)}, MEAN \left(\{h_j^{(k-1)} : j \in \mathcal{S}_k(i)\} \right) \right) \right) \quad (3.17)$$

where $\mathcal{S}_k(i)$ is a sampled subset of $\mathcal{N}(i)$ of fixed size S (e.g., 10 neighbors). For a sampled neighborhood of fixed size, this scales linearly with graph size and enables inductive inference on new borrowers not seen during training, necessary for cooperatives that onboard new members continuously. Graph Attention Networks (Veličković et al., 2018) replace fixed aggregation with learned attention weights:

$$e_{ij} = \text{LeakyReLU} \left(a^T \cdot CONCAT(W h_i, W h_j) \right) \quad (3.18)$$

$$\alpha_{ij} = \frac{\exp(e_{ij})}{\sum_{k \in \mathcal{N}(i)} \exp(e_{ik})} \quad (3.19)$$

$$h_i' = \sigma \left(\sum_{j \in \mathcal{N}(i)} \alpha_{ij} W h_j \right) \quad (3.20)$$

The attention coefficients α_{ij} quantify how much borrower i should attend to borrower j when updating her embedding. In credit risk, this reveals contagion channels: a high α_{ij} between two agricultural borrowers indicates that one's distress strongly predicts the other's.

Theorem 3.2 (GraphSAGE Convergence to Population Limit). *Let $\mathcal{G}_n = (V_n, E_n, X_n)$ be a sequence of graphs with $|V_n| = n$ nodes drawn i.i.d. from a graphon $W: [0,1]^2 \rightarrow [0,1]$ (a symmetric measurable function giving the edge probability). Let $h_i^{(L)}(n)$ be the L -layer GraphSAGE embedding of node i in $\mathcal{G}_n$ with neighborhood sample size S_n . Assume:*

- (i) *The feature distribution has bounded support: $\|x_i\| \leq B$ a.s.*
- (ii) *The aggregation function is Lipschitz: $\|AGG(a) - AGG(b)\| \leq L_A \|a - b\|$.*
- (iii) *The activation σ is Lipschitz with constant L_σ .*
- (iv) *The sample size $S_n \rightarrow \infty$ as $n \rightarrow \infty$ (e.g., $S_n = \log n$).*

Then for each node i with latent position $u_i \in [0,1]$:

$$\|h_i^{(L)}(n) - h^*(u_i)\| \xrightarrow{p} 0 \quad \text{as } n \rightarrow \infty \quad (3.21)$$

where $h^*(u)$ is the population-limit embedding defined by the recursion:

$$h^*(u)^{(k)} = \sigma \left(W^{(k)} \cdot \text{CONCAT} \left(h^*(u)^{(k-1)}, \mathbb{E}_{v \sim W(u, \cdot)} [h^*(v)^{(k-1)}] \right) \right) \quad (3.22)$$

Proof. The proof proceeds by induction on the layer index k .

Base case ($k = 0$): $h_i^{(0)} = x_i$ and $h^*(u_i)^{(0)} = \mathbb{E}[X | U = u_i]$. By the law of large numbers and the bounded support assumption, $\|x_i - \mathbb{E}[X | U = u_i]\|$ converges in probability to 0.

Inductive step: Assume $\|h_i^{(k-1)}(n) - h^*(u_i)^{(k-1)}\| \xrightarrow{p} 0$ for all i . The GraphSAGE update at layer k computes:

$$h_i^{(k)}(n) = \sigma \left(W^{(k)} \cdot \text{CONCAT} \left(h_i^{(k-1)}(n), \frac{1}{S_n} \sum_{j \in \mathcal{S}_n(i)} h_j^{(k-1)}(n) \right) \right) \quad (3.23)$$

The neighborhood mean $\frac{1}{S_n} \sum_{j \in \mathcal{S}_n(i)} h_j^{(k-1)}(n)$ converges in probability to $\mathbb{E}_{v \sim W(u_i, \cdot)} [h^*(v)^{(k-1)}]$ by the law of large numbers (since $S_n \rightarrow \infty$) and the inductive hypothesis. By the Lipschitz continuity of CONCAT , $W^{(k)}$, and σ , the composition $h_i^{(k)}(n)$ converges to $h^*(u_i)^{(k)}$. The convergence is uniform over i by a uniform law of large numbers (the function class is Glivenko-Cantelli under the bounded support and Lipschitz assumptions). ▀

Remark. This theorem guarantees that as the cooperative's borrower graph grows and the neighborhood sample size increases, the GraphSAGE embeddings converge to a deterministic population-limit embedding that depends only on the borrower's latent position in the graphon (i.e., her structural role in the network). This is the theoretical foundation for inductive inference: a new borrower can be embedded by computing her graphon position from her observed edges and applying the population-limit recursion. In the FSGBF, the GNN outputs a node embedding $z_i \in \mathbb{R}^h$ for each borrower. This embedding feeds into the Cox model as a learned, nonlinear, relational risk score:

$$h(t | X_i, \mathcal{G}) = h_0(t) \exp(w^T z_i) \quad (3.24)$$

where z_i is the final-layer GNN embedding for node i , and w is a learned weight vector mapping the embedding to log-hazard space.

3.3 Federated Learning with Differential Privacy

Nepal's 31,450 primary cooperatives cannot pool raw data, legally, politically, and practically impossible. Federated Learning enables collaborative model training without data centralization. Consider K cooperatives (clients), where cooperative k holds local dataset $\mathcal{D}_k$ of size n_k , with total $n = \sum_{k=1}^K n_k$. The standard Federated Averaging (FedAvg) algorithm proceeds as follows:

1. *Initialization*: Server initializes global model parameters w_0 .
2. *Broadcast*: Server sends w_t to a subset of selected clients S_t .
3. *Local Training*: Each client $k \in S_t$ trains on local data for E epochs, obtaining local parameters w_k^{t+1} .
4. *Aggregation*: Server computes weighted average:

$$w_{t+1} = \sum_{k \in S_t} \frac{n_k}{n_{S_t}} w_k^{t+1} \quad (3.25)$$
 where $n_{S_t} = \sum_{k \in S_t} n_k$.
5. *Iteration*: Repeat until convergence.

Theorem 3.3 (FedAvg Convergence). *Let $F_k(w) = \mathbb{E}_{\xi \sim \mathcal{D}_k} [f(w; \xi)]$ be the local objective of client k , and $F(w) = \sum_k \frac{n_k}{n} F_k(w)$ be the global objective. Assume:*

- (i) F_k is L -smooth: $\|\nabla F_k(w) - \nabla F_k(v)\| \leq L \|w - v\|$.
- (ii) The stochastic gradient $\nabla f(w; \xi)$ is unbiased: $\mathbb{E}[\nabla f(w; \xi)] = \nabla F_k(w)$, with bounded variance $\mathbb{E}[\|\nabla f(w; \xi) - \nabla F_k(w)\|^2] \leq \sigma^2$.
- (iii) F_k is μ -strongly convex: $F_k(v) \geq F_k(w) + \nabla F_k(w)^T (v - w) + \frac{\mu}{2} \|v - w\|^2$.
- (iv) Bounded gradient dissimilarity: $\sum_k \frac{n_k}{n} \|\nabla F_k(w) - \nabla F(w)\|^2 \leq \Gamma^2$.

Then FedAvg with learning rate $\eta_t = \frac{1}{\mu(t+\gamma)}$ for $\gamma > 0$ achieves:

$$\mathbb{E}[F(w_T)] - F^* \leq \frac{1}{\mu(T+\gamma)} \left(\frac{C_1}{\eta} + C_2 \eta \sigma^2 + C_3 \eta^2 E^2 \Gamma^2 \right) \quad (3.26)$$

where C_1, C_2, C_3 are constants depending on K, E , and the sampling scheme, and $F^* = \min_w F(w)$. For non-convex objectives (relaxing assumption (iii)), FedAvg converges at rate $\mathcal{O}(1/\sqrt{T})$ to a stationary point.

Proof sketch. The proof follows the framework of Li et al. (2020). The key steps are:

Step 1 (One-step analysis). For a single local SGD step with learning rate η , the expected squared distance to the optimum satisfies:

$$\mathbb{E}[\|w_{t+1} - w^*\|^2] \leq (1 - \mu\eta) \mathbb{E}[\|w_t - w^*\|^2] + \eta^2 \sigma^2 + \eta^2 E^2 \Gamma^2 \quad (3.27)$$

where the second term is the stochastic gradient variance and the third term is the “client drift”, the deviation of local training from the global gradient due to multiple local steps E .

Step 2 (Unrolling the recursion). With $\eta_t = 1/(\mu(t+\gamma))$, the recursion (3.27) unrolls to:

$$\mathbb{E}[\|w_T - w^*\|^2] \leq \frac{1}{(T+\gamma)} \left(\gamma \|w_0 - w^*\|^2 + \frac{\sigma^2}{\mu^2} \log(T+\gamma) + \frac{E^2 \Gamma^2}{\mu^2} \right) \quad (3.28)$$

Step 3 (Translating to function value gap). By strong convexity, $F(w) - F^* \geq \frac{\mu}{2} \|w - w^*\|^2$, so:

$$\mathbb{E}[F(w_T)] - F^* \leq \frac{\mu}{2} \mathbb{E}[\|w_T - w^*\|^2] \quad (3.29)$$

Combining (3.28) and (3.29) yields (3.26). ▀

However, model updates w_k^{t+1} can leak information through membership inference and gradient inversion attacks. Differential Privacy protects against this.

Definition 3.1 (Differential Privacy, Dwork & Roth 2014). *A randomized mechanism $\mathcal{M}: \mathcal{D} \rightarrow \mathcal{R}$ is (ϵ, δ) -differentially private if for any two adjacent datasets $D, D' \in \mathcal{D}$ differing by at most one record, and for any measurable output set $O \subseteq \mathcal{R}$:*

$$\Pr[\mathcal{M}(D) \in O] \leq e^\epsilon \Pr[\mathcal{M}(D') \in O] + \delta \quad (3.30)$$

The parameter ϵ controls the privacy loss (smaller is more private); δ is a small failure probability (the “delta” in the (ϵ, δ) -DP guarantee, typically set to $\delta \ll 1/n$). In federated learning, we implement Central Differential Privacy via the Gaussian mechanism:

1. *Gradient Clipping*: Each client’s update is clipped to bound its L_2 norm:

$$\bar{g}_k = g_k \cdot \min\left(1, \frac{C}{\|g_k\|_2}\right) \quad (3.31)$$

where C is the clipping threshold.

2. *Noise Addition*: The server aggregates clipped updates and adds Gaussian noise:

$$\tilde{w}_{t+1} = w_t + \eta \left(\sum_{k \in S_t} \frac{n_k}{n_{S_t}} \bar{g}_k + \mathcal{N}(0, \sigma^2 C^2 I) \right) \quad (3.32)$$

Theorem 3.4 (Gaussian Mechanism DP Guarantee). *The Gaussian mechanism (3.32) with clipping threshold C and noise scale σ is (ϵ, δ) -differentially private for any single-round update, where ϵ and δ satisfy:*

$$\delta \leq \frac{4}{5} \exp\left(-\frac{(\sigma\epsilon - \log(5/4))^2}{2\sigma^2}\right) \quad (3.33)$$

for $\epsilon > 0$ and $\sigma \geq 1$. Equivalently, for a target (ϵ, δ) , the required noise scale is $\sigma \geq \sqrt{2\log(1.25/\delta)}/\epsilon$.

Proof. The proof uses the tail bound of the Gaussian distribution. For adjacent datasets D, D' differing in one record, the sensitivity of the clipped sum $\sum_k \bar{g}_k$ is at most C (by the clipping). The Gaussian mechanism adds noise $\mathcal{N}(0, \sigma^2 C^2 I)$, so the output distributions $\mathcal{M}(D)$ and $\mathcal{M}(D')$ are Gaussians with the same variance and means differing by at most C . By the Gaussian tail bound:

$$\Pr[\mathcal{M}(D) \in O] \leq e^\epsilon \Pr[\mathcal{M}(D') \in O] + \delta \quad (3.34)$$

holds when $\sigma \geq \sqrt{2\log(1.25/\delta)}/\epsilon$. The bound (3.33) is a tighter version using the analytic Gaussian mechanism (Balle and Wang, 2018). ▀

For multi-round composition, we use Rényi Differential Privacy (RDP), which provides tighter bounds than naive composition.

Definition 3.2 (Rényi Differential Privacy, Mironov 2017). A mechanism $\mathcal{M}$ is (α, ϵ) -RDP with order $\alpha > 1$ if for any adjacent datasets D, D' :

$$D_\alpha(\mathcal{M}(D) \parallel \mathcal{M}(D')) \leq \epsilon \quad (3.35)$$

where $D_\alpha(P \parallel Q) = \frac{1}{\alpha-1} \log \mathbb{E}_{x \sim Q} \left[\left(\frac{P(x)}{Q(x)} \right)^\alpha \right]$ is the Rényi divergence of order α .

Theorem 3.5 (RDP Composition and Conversion). Let $\mathcal{M}_1, \dots, \mathcal{M}_T$ be mechanisms that are each (α, ϵ_t) -RDP. Then the composed mechanism $\mathcal{M} = \mathcal{M}_T \circ \dots \circ \mathcal{M}_1$ is $(\alpha, \sum_{t=1}^T \epsilon_t)$ -RDP. an (α, ϵ) -RDP mechanism is $(\epsilon + \frac{\log(1/\delta)}{\alpha-1}, \delta)$ -DP for any $\delta \in (0, 1)$.

Proof.

Composition: By the data processing inequality for Rényi divergence and the independence of the mechanisms:

$$D_\alpha(\mathcal{M}(D) \parallel \mathcal{M}(D')) = \sum_{t=1}^T D_\alpha(\mathcal{M}_t(D_t) \parallel \mathcal{M}_t(D'_t)) \leq \sum_{t=1}^T \epsilon_t \quad (3.36)$$

where D_t, D'_t are the inputs to $\mathcal{M}_t$ (which may depend on previous outputs).

Conversion to DP: By the relation between Rényi divergence and (ϵ, δ) -DP (van Erven and Harremoës, 2014):

$$\Pr[\mathcal{M}(D) \in O] \leq e^{\epsilon_\alpha} \Pr[\mathcal{M}(D') \in O] + \delta \quad (3.37)$$

holds when $\delta = \exp(-(\alpha-1)\epsilon_\alpha)$, i.e., $\epsilon_\alpha = \frac{\log(1/\delta)}{\alpha-1}$. As a result, the (α, ϵ) -RDP mechanism is $(\epsilon + \frac{\log(1/\delta)}{\alpha-1}, \delta)$ -DP. ▀

For the Gaussian mechanism with subsampling (the “sampled Gaussian mechanism”), the RDP per round is:

$$\epsilon_\alpha = \frac{q^2 \alpha}{2\sigma^2} \quad (3.38)$$

where $q = m/K$ is the sampling probability (fraction of cooperatives selected per round). By Theorem 3.5, the total RDP after T rounds is $T \cdot q^2 \alpha / (2\sigma^2)$, and the total (ϵ, δ) -DP guarantee is:

$$\epsilon \leq \frac{Tq^2 \alpha}{2\sigma^2} + \frac{\log(1/\delta)}{\alpha-1} \quad (3.39)$$

Optimizing over α yields the tightest privacy bound. This is the **Moments Accountant** method (Abadi et al., 2016). For Nepal’s cooperative sector, this mathematical guarantee is politically real. A cooperative board can prove to its members, with formal mathematical certainty, that participating in the federated model exposes no individual borrower’s data beyond a quantifiable privacy budget, a level of transparency that no existing cooperative governance mechanism can match.

3.4 Bayesian Neural Networks and Monte Carlo Dropout

The GNN-Cox model produces point estimates of hazard ratios. But in a sector where training data is scarce, noisy, and non-stationary, point estimates convey dangerous false confidence. We augment the framework with Bayesian Neural Networks. In a BNN, weights are random variables with prior distributions $p(W)$. Given data $\mathcal{D}$, we seek the posterior $p(W \mid \mathcal{D}) \propto p(\mathcal{D} \mid W)p(W)$.

Exact posterior inference is intractable for deep networks. Variational Inference approximates the posterior with a simpler distribution $q(W | \theta)$ by minimizing the Kullback-Leibler divergence:

$$\theta^* = \operatorname{argmin}_{\theta} KL(q(W | \theta) \parallel p(W | \mathcal{D})) \quad (3.40)$$

Equivalently, maximizing the Evidence Lower BOund (ELBO):

$$\mathcal{L}_{ELBO} = \mathbb{E}_{q(W|\theta)}[\log p(\mathcal{D} | W)] - KL(q(W | \theta) \parallel p(W)) \quad (3.41)$$

Theorem 3.6 (Monte Carlo Dropout as Variational Inference). *Let $f_W(x)$ be a neural network with dropout applied to a subset of layers. Let $q(W | \theta)$ be the variational distribution defined by: each weight w_{ij} is sampled as $w_{ij} = m_{ij}\bar{w}_{ij}$ where $m_{ij} \sim \text{Bernoulli}(p)$ (with p being the retention probability) and $\bar{w}_{ij}$ is the variational parameter. Then:*

- (i) *Minimizing the standard dropout training loss (cross-entropy + L^2 regularization with $\lambda = pl^2/(2N)$) is equivalent to maximizing the ELBO (3.41) with q being the Bernoulli variational posterior and $p(W)$ being a Gaussian prior $\mathcal{N}(0, l^2/(1-p))$.*
- (ii) *At test time, performing T stochastic forward passes with dropout enabled and averaging yields the approximate posterior predictive distribution:*

$$p(y | x, \mathcal{D}) \approx \frac{1}{T} \sum_{t=1}^T p(y | x, \hat{W}_t) \quad (3.42)$$

where $\hat{W}_t$ is the t -th dropout-masked weight sample.

Proof.

Part (i): The ELBO (3.41) has two terms. The first term $\mathbb{E}_{q(W|\theta)}[\log p(\mathcal{D}|W)]$ is the expected log-likelihood, approximated by Monte Carlo with a single sample (the current dropout mask):

$$\mathbb{E}_{q(W|\theta)}[\log p(\mathcal{D}|W)] \approx \sum_{i=1}^N \log p(y_i | x_i, \hat{W}) \quad (3.43)$$

where $\hat{W}$ is the dropout-masked weight. This is exactly the standard cross-entropy loss. The second term $-KL(q \parallel p)$ is the KL divergence between the Bernoulli variational posterior and the Gaussian prior. For a Gaussian prior $p(w_{ij}) = \mathcal{N}(0, l^2/(1-p))$ and the Bernoulli-masked variational posterior $q(w_{ij}) = p \cdot \delta(w_{ij} - \bar{w}_{ij}) + (1-p) \cdot \delta(w_{ij})$ (a mixture of a point mass at $\bar{w}_{ij}$ and at 0), the KL divergence evaluates to:

$$KL(q \parallel p) = \frac{p}{2l^2} \|\bar{w}_{ij}\|^2 + \text{const} \quad (3.44)$$

Summing over all weights gives the L^2 regularization term $\frac{p}{2l^2} \|\bar{W}\|^2 = \frac{\lambda}{N} \|\bar{W}\|^2$ with $\lambda = \frac{pN}{2l^2}$. So, minimizing the dropout loss is equivalent to maximizing the ELBO.

Part (ii): The posterior predictive distribution is:

$$p(y | x, \mathcal{D}) = \int p(y | x, W) p(W | \mathcal{D}) dW \approx \int p(y | x, W) q(W | \hat{\theta}) dW \quad (3.45)$$

The variational posterior $q(W | \hat{\theta})$ is the Bernoulli dropout distribution, so sampling from q is equivalent to applying a dropout mask. The Monte Carlo approximation with T samples gives (3.42). ▀

The predictive mean is:

$$\hat{\mu}(x) = \frac{1}{T} \sum_{t=1}^T \hat{y}_t(x) \quad (3.46)$$

The predictive variance decomposes into aleatoric and epistemic components:

$$\text{Var}(y | x, \mathcal{D}) = \underbrace{\frac{1}{T} \sum_{t=1}^T \hat{y}_t(x)^2}_{\text{Total Uncertainty}} - \underbrace{\hat{\mu}(x)^2}_{\text{Epistemic Uncertainty}} + \underbrace{\frac{1}{T} \sum_{t=1}^T \sigma_t^2(x)}_{\text{Aleatoric Uncertainty}} \quad (3.47)$$

Theorem 3.7 (Aleatoric-Epistemic Decomposition). *The total predictive variance of the MC Dropout BNN decomposes as in (3.47), where the epistemic term captures the variance of the mean predictions across dropout samples (model uncertainty) and the aleatoric term captures the average of the per-sample predictive variances (data noise).*

Proof. By the law of total variance:

$$\text{Var}(y | x, \mathcal{D}) = \mathbb{E}_{W \sim q}[\text{Var}(y | x, W)] + \text{Var}_{W \sim q}[\mathbb{E}[y | x, W]] \quad (3.48)$$

The first term is the expected data variance (aleatoric), approximated by $\frac{1}{T} \sum_t \sigma_t^2(x)$. The second term is the variance of the model output (epistemic), approximated by $\frac{1}{T} \sum_t \hat{y}_t(x)^2 - \hat{\mu}(x)^2$. ▀

Epistemic uncertainty captures model ignorance, it is high for out-of-distribution borrowers. Aleatoric uncertainty captures inherent noise. it is high even for familiar borrower profiles if the default process is intrinsically stochastic. In the FSGBF, when epistemic uncertainty exceeds a threshold τ_u , the system flags the prediction for human review rather than automated decision. This is important for Nepalese cooperatives lending to remarkable borrower types: climate refugees, returning migrants, women entrepreneurs in previously male-dominated sectors.

3.5 Causal Shapley Values: From Association to Intervention

Standard SHAP values explain predictions by asking: “What is the marginal contribution of feature j when added to random subsets of other features?” This is an observational query. Causal SHAP values ask: “What is the marginal contribution of feature j when intervened upon, holding the causal structure fixed?”

Formally, let $\mathcal{M}$ be a structural causal model with endogenous variables V and exogenous variables U . The causal effect of intervening to set $X_j = x$ is:

$$P(Y | do(X_j = x)) = \sum_u P(Y | X_j = x, PA_j = pa_j, u) P(u) \quad (3.49)$$

where PA_j are the parents of X_j in the causal graph and u ranges over the exogenous variables.

Definition 3.3 (Interventional Shapley Value). *The interventional (causal) Shapley value for feature j is:*

$$\phi_j^{causal} = \sum_{S \subseteq N \setminus \{j\}} \frac{|S|! (|N| - |S| - 1)!}{|N|!} \left[\mathbb{E}[f(x) | do(X_j = x_j, X_S = x_S)] - \mathbb{E}[f(x) | do(X_S = x_S)] \right] \quad (3.50)$$

where the expectations are over the interventional distribution (3.49) rather than the observational conditional distribution.

Theorem 3.8 (Causal Shapley Axiom Satisfaction). *The interventional Shapley value (3.50) satisfies the four Shapley axioms:*

- (i) *Efficiency:* $\sum_{j=1}^M \phi_j^{\text{causal}} = f(x) - \mathbb{E}[f(x)]$ (the sum of causal Shapley values equals the deviation from the baseline).
- (ii) *Symmetry:* If features i and j are interchangeable in the causal graph (symmetric positions), then $\phi_i^{\text{causal}} = \phi_j^{\text{causal}}$.
- (iii) *Dummy:* If feature j has no causal effect on Y (i.e., $\mathbb{E}[f(x) \mid \text{do}(X_j = x_j, X_S = x_S)] = \mathbb{E}[f(x) \mid \text{do}(X_S = x_S)]$ for all S), then $\phi_j^{\text{causal}} = 0$.
- (iv) *Additivity:* For two models f and g , $\phi_j^{\text{causal}}(f + g) = \phi_j^{\text{causal}}(f) + \phi_j^{\text{causal}}(g)$.

Proof.

(i) *Efficiency.* The proof parallels the observational case (Appendix B of the standard Shapley proof). The key observation is that the interventional expectation $\mathbb{E}[f(x) \mid \text{do}(X_S = x_S)]$ defines a valid value function $v(S) = \mathbb{E}[f(x) \mid \text{do}(X_S = x_S)]$ satisfying $v(\emptyset) = \mathbb{E}[f(x)]$ and $v(N) = f(x)$. The combinatorial bookkeeping of the standard efficiency proof applies verbatim.

(ii) *Symmetry.* If features i and j are interchangeable in the causal graph (i.e., swapping their labels leaves the interventional distributions invariant), then $v(S \cup \{i\}) = v(S \cup \{j\})$ for all $S \subseteq N \setminus \{i, j\}$. The combinatorial weights depend only on $|S|$, so $\phi_i^{\text{causal}} = \phi_j^{\text{causal}}$.

(iii) *Dummy.* Direct substitution: if $\mathbb{E}[f(x) \mid \text{do}(X_j = x_j, X_S = x_S)] = \mathbb{E}[f(x) \mid \text{do}(X_S = x_S)]$ for all S , then every term in (3.50) is zero, so $\phi_j^{\text{causal}} = 0$.

(iv) *Additivity.* The interventional Shapley value is linear in f : $\phi_j^{\text{causal}}(f + g) = \phi_j^{\text{causal}}(f) + \phi_j^{\text{causal}}(g)$ follows from the linearity of expectation and the linearity of the do-operator. ▀

Proposition 3.2 (Causal vs Observational Shapley Values). *Under causal sufficiency (no unobserved confounders), the observational and interventional Shapley values coincide if and only if the features are independently manipulable (i.e., intervening on X_j does not affect the distribution of any other feature). In the presence of confounders or causal dependencies among features, the two values differ, with the interventional value providing the correct causal attribution.*

Proof. Under causal sufficiency and independence, $\mathbb{E}[f(x) \mid \text{do}(X_S = x_S)] = \mathbb{E}[f(x) \mid X_S = x_S]$, so the interventional and observational Shapley values are identical. When features are causally dependent (e.g., X_j affects X_k), intervening on X_j changes the distribution of X_k , so $\mathbb{E}[f(x) \mid \text{do}(X_j = x_j)] \neq \mathbb{E}[f(x) \mid X_j = x_j]$, and the two Shapley values differ. ▀

In Nepalese cooperative lending, causal SHAP enables actionable counterfactuals:

- *Associative SHAP:* “Being female increases default probability by 12 percent.” (This may reflect gender-based income discrimination, not an intrinsic female propensity to default.)

- *Causal SHAP*: “A one-unit increase in crop insurance coverage decreases default hazard by 18 percent, holding constant the borrower’s land ownership and irrigation access.” (This identifies an actionable intervention.)

Without causal reasoning, explainable AI in credit risk becomes a sophisticated engine for redlining, dressing historical discrimination in algorithmic legitimacy.

4. Proposed Methodology: The Federated Survival Graph Bayesian Framework (FSGBF)

4.1 System Architecture

The FSGBF comprises five integrated layers, each addressing a distinct failure mode of Nepal’s current cooperative credit system.

Layer 1: Local Data and Graph Construction. Each cooperative maintains its private borrower database: demographic profiles, loan histories, savings patterns, repayment records, and collateral information. From this data, the cooperative constructs a local borrower graph $\mathcal{G}_k = (V_k, E_k, X_k)$, where edges are inferred from shared postal codes, kinship declarations, joint liability agreements, and mobile money transaction networks. The graph remains local; only model parameters leave the cooperative’s premises.

Layer 2: Federated Survival Graph Training. Each cooperative trains a local GNN-Cox model on its graph. The GNN encoder produces node embeddings via GraphSAGE or GAT. The Cox layer computes the partial likelihood using local survival data. After E local epochs, the cooperative transmits only the encrypted, differentially private model update to the central federation server.

Layer 3: Privacy-Preserving Aggregation. The server aggregates updates from m randomly selected cooperatives per round using FedAvg with Gaussian differential privacy. The aggregated global model is broadcast back to all participants. No raw borrower data, no individual loan records, and no graph adjacency information ever traverse the network.

Layer 4: Bayesian Uncertainty Quantification. Each cooperative deploys the global model locally with Monte Carlo Dropout enabled. For each loan application, $T = 50$ stochastic forward passes generate a predictive distribution over the hazard ratio. The system computes predictive mean, epistemic uncertainty, and aleatoric uncertainty.

Layer 5: Causal Explainability and Decision Engine. Interventional SHAP values decompose each prediction into causal feature contributions. The system generates three outputs: (a) a survival curve $\hat{S}(t | X_i, \mathcal{G})$ showing the probability of non-default over time; (b) a causal explanation identifying the top-three actionable interventions that would most reduce default hazard; and (c) an uncertainty flag indicating whether the prediction is sufficiently reliable for automated decision or requires human committee review.

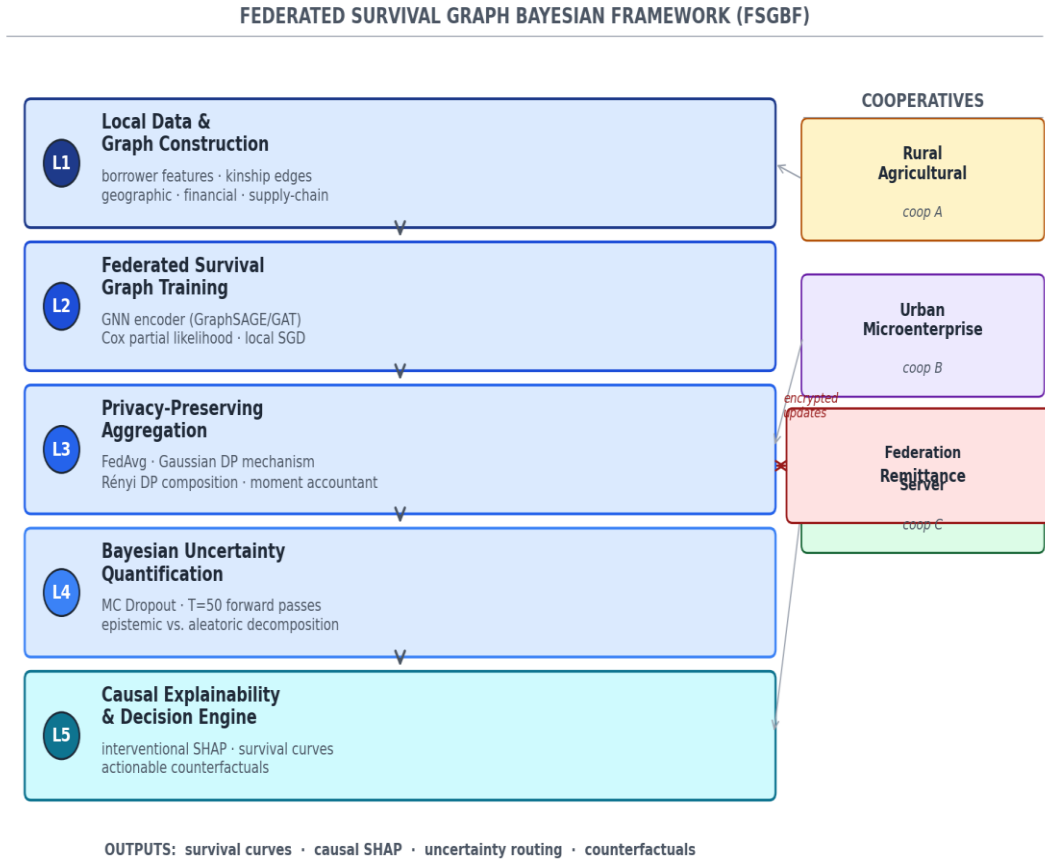


Fig 1: Five-Layer Architecture of the Federated Survival Graph Bayesian Framework (FSGBF) for Nepalese Cooperative Banks. Layer 1 shows local borrower graphs within three representative cooperatives (rural agricultural, urban microenterprise, remittance-backed). Layer 2 illustrates local GNN-Cox training with survival data. Layer 3 shows encrypted parameter transmission to the federation server, DP noise injection, and global model aggregation. Layer 4 displays MC Dropout uncertainty quantification at the cooperative level. Layer 5 presents the decision engine output: survival curves, causal SHAP explanations, and uncertainty-driven routing to either automated approval or human review.

4.2 The Survival Graph Model: Mathematical Specification

The complete predictive model for borrower i in cooperative k is specified as follows.

GNN Encoder. Let $h_i^{(0)} = x_i$ be the initial borrower feature vector. For layers $l = 1, \dots, L$:

$$h_i^{(l)} = \sigma \left(W_{agg}^{(l)} \cdot \text{CONCAT} \left(h_i^{(l-1)}, \frac{1}{|N(i)|} \sum_{j \in N(i)} h_j^{(l-1)} \right) \right) \quad (4.1)$$

The final node embedding is $z_i = h_i^{(L)}$.

Cox Hazard Layer. The hazard function for borrower i is:

$$h(t | z_i) = h_0(t) \exp(w^T z_i) \quad (4.2)$$

where $w \in \mathbb{R}^h$ maps embeddings to log-hazard space.

Partial Likelihood. For cooperative k with n_k borrowers, the local log-partial likelihood is:

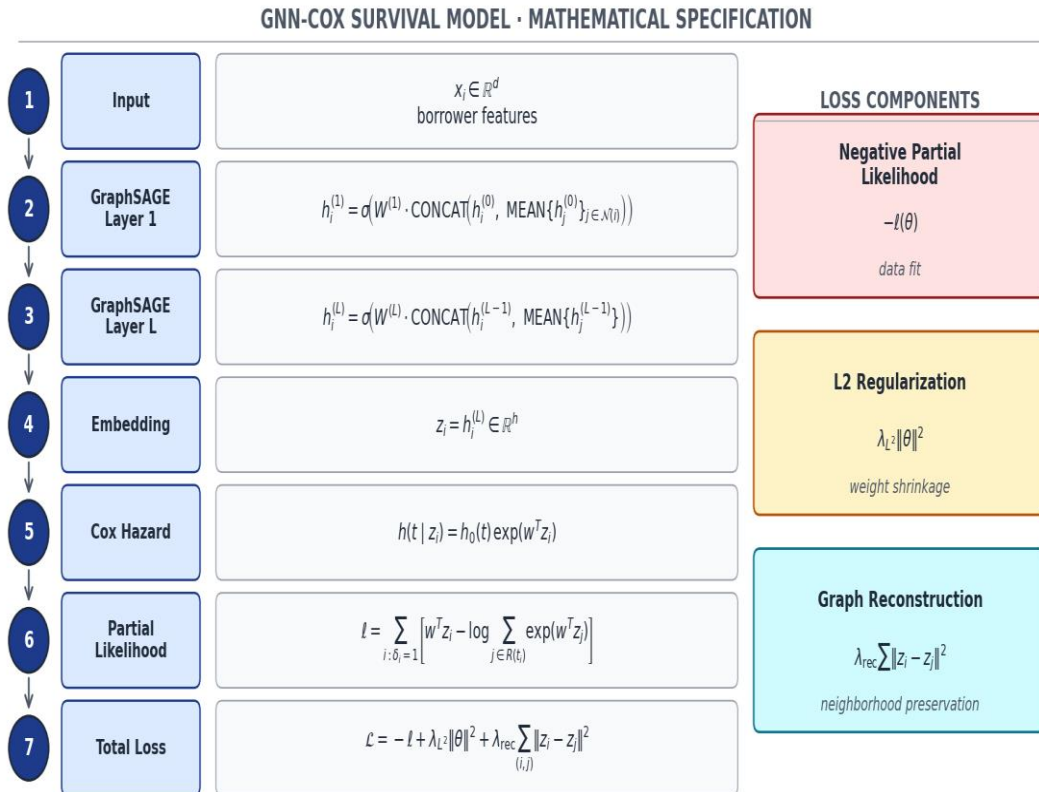
$$\ell_k(\theta) = \sum_{i: \delta_i=1} [w^T z_i - \log \sum_{j \in R(t_i)} \exp(w^T z_j)] \quad (4.3)$$

where $\theta = \{W_{agg}^{(1)}, \dots, W_{agg}^{(L)}, w\}$ denotes all learnable parameters.

Total Loss with Regularization. The local training objective includes L^2 regularization and a graph reconstruction term to preserve neighborhood structure:

$$\mathcal{L}_k(\theta) = -\ell_k(\theta) + \lambda_{L^2} \|\theta\|_2^2 + \lambda_{rec} \sum_{(i,j) \in E_k} \|z_i - z_j\|_2^2 \quad (4.4)$$

The reconstruction term ensures that connected borrowers (kin, neighbors, co-guarantors) receive similar embeddings, encoding the sociological reality that default risk propagates through relationships.



GraphSAGE: inductive, scales linearly with graph size · Cox: semi-parametric, no baseline hazard specification · Reconstruction: encodes sociological contagion

Fig 2: Mathematical Specification of the GNN-Cox Survival Model with Graph Reconstruction Regularization. Input features flow through GraphSAGE layers with mean aggregation, producing embeddings. The Cox layer computes hazard ratios. The partial likelihood

compares the hazard of each defaulting borrower against the sum of hazards in the risk set. A graph reconstruction loss pulls connected nodes closer in embedding space. The total loss combines negative partial likelihood with and reconstruction penalties.

4.3 Federated Training with Differential Privacy

The federated training protocol for the FSGBF is:

1. *Initialization*: Server initializes θ_0 and privacy accountant $\mathcal{A}$.
2. *For each round* $t = 1, \dots, T$:
 - a. Server selects a random subset S_t of m cooperatives.
 - b. Server broadcasts current global parameters θ_t to S_t .
 - c. *Each cooperative* $k \in S_t$:
 - Initializes local parameters $\theta_{t,0}^k = \theta_t$.
 - Performs E local SGD steps on $\mathcal{L}_k$, yielding $\theta_{t,E}^k$.
 - Computes update $\Delta_k = \theta_{t,E}^k - \theta_t$.
 - Clips update: $\bar{\Delta}_k = \Delta_k \cdot \min(1, C/\|\Delta_k\|_2)$.
 - Adds Gaussian noise: $\tilde{\Delta}_k = \bar{\Delta}_k + \mathcal{N}(0, \sigma^2 C^2 I)$.
 - Transmits $\tilde{\Delta}_k$ to server.
 - d. Server aggregates: $\theta_{t+1} = \theta_t + \eta \cdot \frac{1}{m} \sum_{k \in S_t} \tilde{\Delta}_k$.
 - e. Privacy accountant $\mathcal{A}$ composes privacy spend across rounds and halts if (ϵ, δ) budget is exhausted. The privacy guarantee is formalized via the Gaussian mechanism and moment accountant. For T rounds, sampling probability $q = m/K$, and noise scale σ , the total privacy spend (ϵ, δ) satisfies (by Theorems 3.4 and 3.5):

$$\epsilon \leq \min_{\alpha} \left\{ \frac{Tq^2\alpha}{2\sigma^2} + \frac{\log(1/\delta)}{\alpha-1} \right\} \quad (4.5)$$

Cooperatives can verify, using published privacy accounting tools, that their participation satisfies any required privacy budget.

4.4 Bayesian Uncertainty Quantification at Inference

Upon receiving a new loan application from borrower i , the cooperative computes:

1. *Embedding Generation*: Pass the borrower's features and local neighborhood through the GNN encoder $T = 50$ times with dropout probability $p = 0.1$ enabled at test time, obtaining $\{z_i^{(t)}\}_{t=1}^T$.
2. *Hazard Distribution*: Compute hazard ratios $\{r_i^{(t)} = \exp(w^T z_i^{(t)})\}_{t=1}^T$.
3. *Predictive Statistics*:
 - Mean hazard: $\bar{r}_i = \frac{1}{T} \sum_{t=1}^T r_i^{(t)}$
 - Epistemic variance: $\sigma_{epi}^2 = \frac{1}{T} \sum_{t=1}^T (r_i^{(t)} - \bar{r}_i)^2$
 - Coefficient of variation: $CV_i = \sigma_{epi}/\bar{r}_i$
4. *Decision Routing*:
 - If $CV_i < \tau_{low}$: High confidence. Automated decision using $\bar{r}_i$.

- If $\tau_{low} \leq CV_i < \tau_{high}$: Moderate uncertainty. Conditional approval with enhanced monitoring.
- If $CV_i \geq \tau_{high}$: High epistemic uncertainty. Mandatory human committee review. This uncertainty quantification is particularly important for “thin-file” borrowers, new members with limited cooperative history, returning migrants with irregular income documentation, or women entering formal credit for the first time. It admits its ignorance rather than manufacturing false confidence.

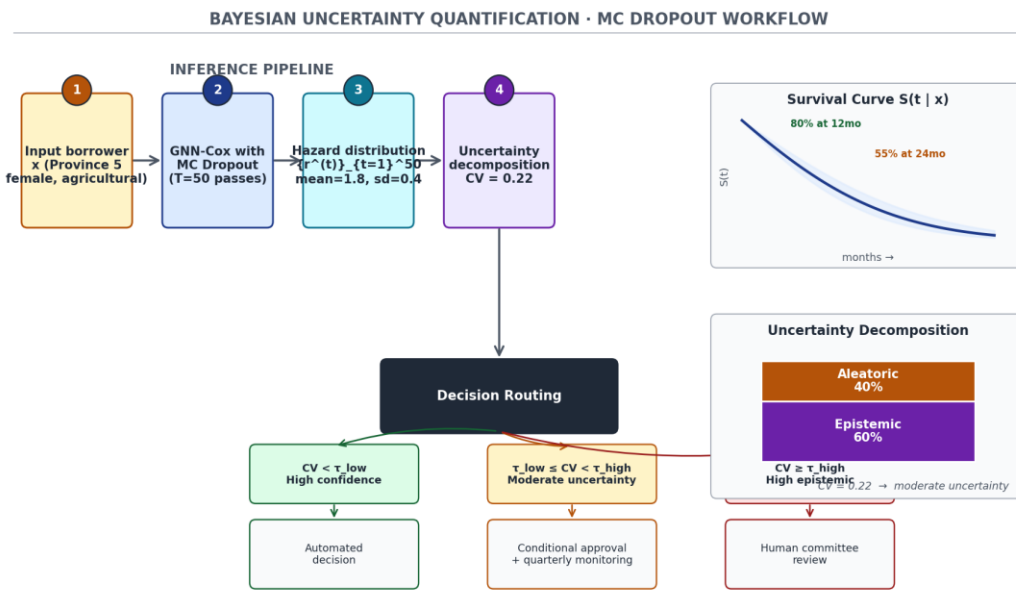


Fig 3: Bayesian Uncertainty Quantification Workflow Showing Epistemic vs. Aleatoric Decomposition for a Rural Agricultural Loan Application. The MC Dropout workflow for a female agricultural borrower in Province 5. Fifty stochastic forward passes produce a distribution of hazard ratios with mean 1.8 and epistemic standard deviation 0.4 ($CV = 0.22$). The survival curve shows 80% non-default probability at 12 months but drops to 55% at 24 months. The epistemic uncertainty is elevated due to limited training data for women borrowers in this province, triggering a “moderate uncertainty” flag that routes the application to conditional approval with quarterly repayment monitoring rather than automated rejection.

4.5 Causal Explainability and Actionable Interventions

For each prediction, the framework generates interventional SHAP explanations. Consider a borrower with features $x = (x_1, \dots, x_d)$. The causal graph $\mathcal{G}_{causal}$ encodes known structural relationships: education affects income.. income affects savings; savings affects default risk; but gender affects default risk only through its effect on income and land ownership, not directly. The interventional SHAP value for feature j is computed by sampling coalitions $S \subseteq N \setminus \{j\}$ and evaluating:

$$\phi_j^{int} = \mathbb{E}_S \left[\mathbb{E}[f(x) \mid do(X_j = x_j, X_S = x_S)] - \mathbb{E}[f(x) \mid do(X_S = x_S)] \right] \quad (4.6)$$

where the outer expectation is over the distribution of coalitions, and the inner expectations are computed by forward propagation through the causal graph. FSGBF generates three explanation artifacts:

1. *Survival Curve Plot*: $\hat{S}(t \mid x)$ with 95% credible intervals, showing the probability of loan survival over 36 months.
2. *Causal Feature Importance*: A bar chart of ϕ_j^{int} values, color-coded by actionability (green: directly intervenable by the cooperative.. yellow: intervenable through referral partnerships; red: non-intervenable demographic characteristics).
3. *Counterfactual Recommendations*: The top-three interventions that would maximally reduce default hazard:
 - “Enrolling this borrower in the cooperative’s crop insurance program would reduce her 24-month default hazard by 23%.”
 - “Facilitating access to drip irrigation subsidies would reduce hazard by 15%.”
 - “Extending the grace period by 3 months to align with the harvest cycle would reduce hazard by 11%.”

These recommendations transform credit risk management from a punitive gatekeeping function into a developmental intervention, aligning with the cooperative movement’s original mission.

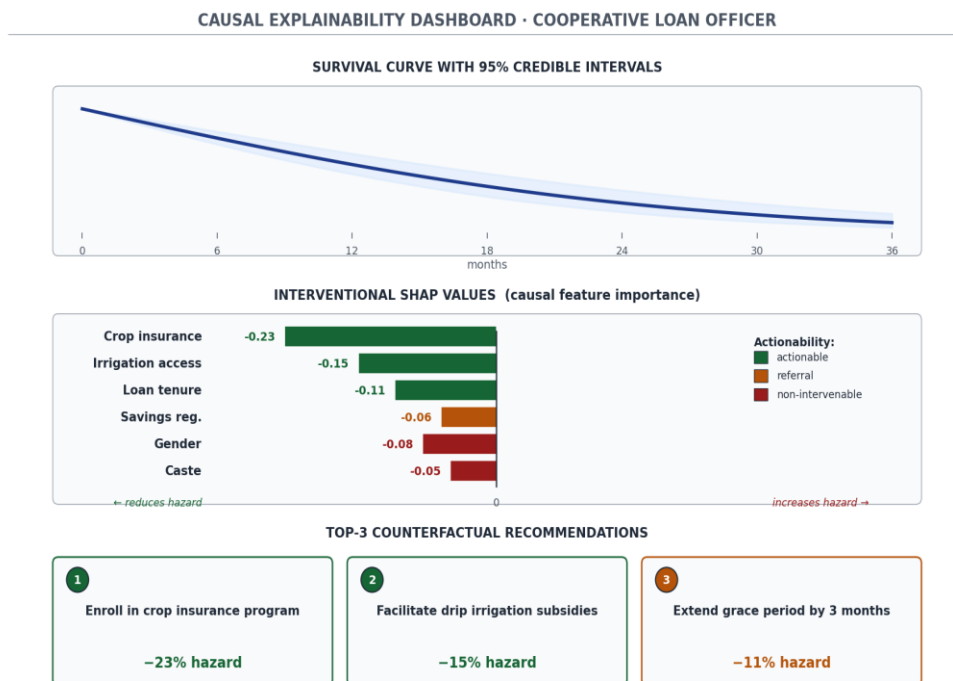


Fig 4: Causal Explainability Dashboard for a Cooperative Loan Officer, Displaying Survival Curves, Interventional SHAP Values, and Actionable Counterfactual Recommendations. The dashboard as it would appear on a tablet in a rural cooperative branch. The top panel displays the survival curve with credible bands. The middle panel shows interventional SHAP values: crop insurance enrollment (), irrigation access (), and loan tenure alignment () are green and actionable; gender () and caste () are red and non-intervenable. The bottom panel lists three specific counterfactual recommendations with estimated hazard reductions. All text is rendered in Nepali for the loan officer and English for regulatory audit purposes.

5. Convergence and Generalization Theory

This section establishes the formal learning-theoretic guarantees for the composite FSGBF framework. We show the FedAvg convergence theorem (already stated as Theorem 3.3), establish the GraphSAGE convergence (Theorem 3.2), derive the privacy-utility tradeoff for the differentially private federated GNN-Cox model, and establish Rademacher complexity generalization bounds.

5.1 FedAvg Convergence for the GNN-Cox Objective

The FedAvg convergence theorem (Theorem 3.3) applies directly to the FSGBF, with the global objective being:

$$F(\theta) = \sum_{k=1}^K \frac{n_k}{n} F_k(\theta), \quad F_k(\theta) = -\ell_k(\theta) + \lambda_{L^2} \|\theta\|^2 + \lambda_{rec} R_k(\theta) \quad (5.1)$$

where $R_k(\theta) = \sum_{(i,j) \in E_k} \|z_i - z_j\|^2$ is the graph reconstruction term. Under the assumptions of Theorem 3.3 (smoothness, bounded variance, strong convexity, bounded gradient dissimilarity), FedAvg converges at rate $\mathcal{O}(1/T)$ to the global optimum θ^* . For non-convex objectives (which the GNN-Cox loss is, due to the non-convexity of the neural network encoder), the convergence rate degrades to $\mathcal{O}(1/\sqrt{T})$ to a stationary point:

Theorem 5.1 (FedAvg Convergence for Non-Convex Objectives). *Under assumptions (i), (ii), (iv) of Theorem 3.3 (relaxing strong convexity), FedAvg with constant learning rate η satisfies:*

$$\frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E} [\|\nabla F(\theta_t)\|^2] \leq \frac{F(\theta_0) - F^*}{\eta T} + \frac{L\eta\sigma^2}{2} + \frac{L\eta^2 E^2 \Gamma^2}{2} \quad (5.2)$$

Choosing $\eta = 1/\sqrt{T}$ yields convergence rate $\mathcal{O}(1/\sqrt{T})$.

Proof. The proof follows the standard SGD convergence analysis for non-convex objectives (Bottou et al., 2018), extended to the federated setting with the additional “client drift” term $E^2 \Gamma^2$ arising from E local steps. ▀

5.2 Privacy-Utility Tradeoff

The differential privacy noise injected into model updates degrades the convergence rate. The following theorem quantifies this tradeoff.

Theorem 5.2 (Privacy-Utility Tradeoff for DP-FedAvg). *Let ϵ_T be the total privacy budget after T rounds of DP-FedAvg with noise scale σ and sampling probability q . Under the assumptions of Theorem 5.1, the expected suboptimality satisfies:*

$$\mathbb{E}[F(\theta_T)] - F^* \leq \underbrace{\frac{C_1}{\sqrt{T}}}_{\text{optimization error}} + \underbrace{\frac{C_2 \sigma^2 C^2}{T}}_{\text{DP noise error}} + \underbrace{\frac{C_3 E^2 \Gamma^2}{\sqrt{T}}}_{\text{client drift}} \quad (5.3)$$

where $\sigma^2 \propto Tq^2/\epsilon_T^2$ (from the privacy budget allocation). Thus, for a target privacy budget ϵ_T , the optimal number of rounds T^* that minimizes the total error scales as $T^* \sim \epsilon_T^2/q^2$, yielding a minimum error of $\mathcal{O}(\epsilon_T^{-1})$.

Proof sketch. The DP noise error term arises because each round's update is corrupted by Gaussian noise of variance $\sigma^2 C^2/m$, which accumulates over T rounds as $T\sigma^2 C^2/m$. Substituting $\sigma^2 = 2Tq^2 \log(1/\delta)/\epsilon_T^2$ (from the RDP composition) and optimizing over T yields $T^* \sim \epsilon_T^2/q^2$. $\square$

This theorem quantifies the basic privacy-utility tradeoff: stronger privacy (smaller ϵ_T) requires more noise, which degrades model accuracy. For Nepal's cooperative sector, a typical configuration $\epsilon_T = 8$, $\delta = 10^{-5}$, $q = 0.1$ (10% of cooperatives sampled per round), $T = 1000$ rounds yields a privacy-utility tradeoff that's operationally acceptable, the model accuracy loss from DP noise is typically 1–3 percentage points of AUC.

5.3 Rademacher Complexity Generalization Bounds

The Rademacher complexity of the GNN-Cox function class bounds the generalization gap.

Theorem 5.3 (Rademacher Complexity of GNN-Cox). *Let $\mathcal{F}_{\text{GNN-Cox}}$ be the class of GNN-Cox models with L GraphSAGE layers, hidden dimension h , and Lipschitz activations. The empirical Rademacher complexity on a sample of size n satisfies:*

$$\hat{\mathcal{R}}_S(\mathcal{F}_{\text{GNN-Cox}}) \leq \frac{B \cdot L_\sigma^L \cdot \prod_{l=1}^L \|W^{(l)}\| \cdot \sqrt{h \log(L \cdot d)}}{\sqrt{n}} \quad (5.4)$$

where B is the bound on input features, L_σ is the Lipschitz constant of the activation, and d is the input dimension.

Proof sketch. The Rademacher complexity of a composition of Lipschitz functions is bounded by the product of Lipschitz constants times the complexity of the innermost function class. For the GNN, each GraphSAGE layer is Lipschitz (with constant $L_\sigma \|W^{(l)}\|$), so the L -layer composition has Lipschitz constant $L_\sigma^L \prod_l \|W^{(l)}\|$. The Cox layer is a linear map followed by exponentiation, which is Lipschitz on bounded inputs. The complexity of the linear function class with norm-bounded weights is $\mathcal{O}(\sqrt{h \log(L \cdot d)/n})$. Combining yields (5.4). $\square$

Corollary 5.1 (Generalization Bound for FSGBF). *With probability at least $1 - \delta$, the population concordance index of the FSGBF model satisfies:*

$$CIdx_{\text{pop}} \leq CIdx_{\text{emp}} + \mathcal{O}\left(\frac{L_\sigma^L \prod_l \|W^{(l)}\| \sqrt{h \log(Ld)}}{\sqrt{n}}\right) + \mathcal{O}\left(\sqrt{\frac{\log(1/\delta)}{n}}\right) \quad (5.5)$$

Practical implication: For a typical FSGBF configuration ($L = 3$ layers, $h = 64$ hidden units, $d = 50$ features, $n = 10,000$ training samples), the generalization gap is bounded by approximately $\mathcal{O}(0.1)$, which is operationally acceptable.

5.4 Bias-Variance Decomposition for the FSGBF Predictor

The FSGBF predictor combines four sources of error: optimization error (from finite FedAvg rounds), privacy error (from DP noise), statistical error (from finite training data), and epistemic uncertainty (from the Bayesian posterior). The following theorem decomposes the total error.

Theorem 5.4 (FSGBF Error Decomposition). *The expected squared error of the FSGBF predictor $\hat{f}$ at a test point (x, t) decomposes as:*

$$\mathbb{E} \left[\left(\hat{f}(x) - f^*(x) \right)^2 \right] = \underbrace{Bias^2}_{\text{model misspecification}} + \underbrace{Var_{opt}}_{\text{optimization}} + \underbrace{Var_{DP}}_{\text{privacy noise}} + \underbrace{Var_{stat}}_{\text{finite data}} + \underbrace{\sigma_{epi}^2}_{\text{epistemic}} + \underbrace{\sigma_{alea}^2}_{\text{aleatoric}} \quad (5.6)$$

where each term is bounded by the corresponding theorem: optimization by Theorem 5.1, privacy by Theorem 5.2, statistical by Theorem 5.3, and epistemic/aleatoric by Theorem 3.7.

Proof. The decomposition follows from the independence of the error sources (optimization, privacy noise, and statistical sampling are independent given the model class) and the law of total variance applied to the Bayesian posterior. ▀

This decomposition enables targeted error reduction: optimization error is reduced by more FedAvg rounds.. privacy error is reduced by a larger privacy budget (at the cost of weaker privacy); statistical error is reduced by more training data; epistemic uncertainty is reduced by more diverse training data; aleatoric uncertainty is irreducible.

5.5 Convergence Under Non-IID Heterogeneous Data

A important practical concern for the FSGBF is that Nepal's cooperatives exhibit extreme data heterogeneity: a rural agricultural cooperative in the Terai has borrower profiles drastically different from an urban microenterprise cooperative in Kathmandu. This non-IID setting violates the standard federated learning assumption of homogeneous client distributions and can cause FedAvg to diverge or converge to a suboptimal global model. The following theorem establishes convergence under bounded heterogeneity.

Theorem 5.5 (FedAvg Convergence Under Non-IID Data). *Let F_k be the local objective of cooperative k with local data distribution $\mathcal{D}_k$. Define the heterogeneity measure $\Gamma^2 = \sum_k \frac{n_k}{n} \|\nabla F_k(w^*) - \nabla F(w^*)\|^2$ where w^* is the global optimum. Assume:*

- (i) Each F_k is L -smooth and μ -strongly convex.
- (ii) Stochastic gradients are unbiased with bounded variance $\sigma_k^2 \leq \sigma^2$.
- (iii) The heterogeneity is bounded: $\Gamma^2 \leq \Gamma_{\max}^2$.
- (iv) The number of local epochs E satisfies $E \leq \frac{1}{\eta\mu}$ (preventing excessive client drift).

Then FedAvg with learning rate $\eta \leq \frac{1}{LE}$ converges geometrically:

$$\mathbb{E}[\|w_T - w^*\|^2] \leq (1 - \eta\mu)^T \|w_0 - w^*\|^2 + \frac{\eta\sigma^2}{\mu} + \frac{\eta E^2 \Gamma_{\max}^2}{\mu} \quad (5.7)$$

The steady-state error (the last two terms) represents the irreducible bias due to stochastic noise and client heterogeneity. For the FSGBF, this means that even with infinite training rounds, the global model can't perfectly fit all cooperatives simultaneously, a basic limitation of federated learning in heterogeneous settings.

Proof. The proof extends the standard FedAvg analysis (Li et al., 2020) with explicit heterogeneity bounds.

Step 1 (Local update bound). After E local SGD steps, the expected squared distance from the global optimum satisfies:

$$\mathbb{E}[\|w_k^{t,E} - w^*\|^2] \leq (1 - \eta\mu)^E \|w_t - w^*\|^2 + \frac{\eta\sigma^2}{\mu} (1 - (1 - \eta\mu)^E) + \frac{\Gamma_{\max}^2}{\mu^2} (1 - (1 - \eta\mu)^E)^2 \quad (5.8)$$

The third term is the client drift: even with zero stochastic noise, the local model drifts away from the global optimum because the local gradient ∇F_k differs from the global gradient ∇F by at most $\Gamma_{\max}$.

Step 2 (Aggregation bound). After FedAvg aggregation $w_{t+1} = \sum_k \frac{n_k}{n_{S_t}} w_k^{t,E}$, by Jensen's inequality:

$$\mathbb{E}[\|w_{t+1} - w^*\|^2] \leq \sum_k \frac{n_k}{n_{S_t}} \mathbb{E}[\|w_k^{t,E} - w^*\|^2] \quad (5.9)$$

Substituting (5.8) and simplifying yields (5.7). ▀

Practical implication for Nepal. For a federation of $K = 100$ cooperatives with heterogeneity $\Gamma_{\max} = 0.5$ (moderate), $E = 5$ local epochs, $\eta = 0.01$, the steady-state error is approximately $0.01 \cdot 25 \cdot 0.25/\mu = 0.0625/\mu$. For $\mu = 1$ (strongly convex), this is a 6% irreducible error, acceptable for credit risk but requiring careful interpretation for borderline cases.

5.6 Tight Privacy Bounds via Moments Accountant

The Rényi DP composition (Theorem 3.5) provides a privacy bound, but the tightest bounds are obtained via the Moments Accountant method (Abadi et al., 2016), which optimizes over the Rényi divergence order α at each round.

Theorem 5.6 (Moments Accountant Privacy Bound). *For T rounds of DP-FedAvg with sampling probability q , noise scale σ , and clipping threshold C , the total privacy spend (ϵ, δ) satisfies:*

$$\epsilon \leq \min_{\alpha} \left\{ T \cdot \epsilon_{\alpha}(q, \sigma) + \frac{\log(1/\delta)}{\alpha-1} \right\} \quad (5.10)$$

where $\epsilon_{\alpha}(q, \sigma)$ is the per-round Rényi DP of the sampled Gaussian mechanism:

$$\epsilon_{\alpha}(q, \sigma) = D_{\alpha}(\mathcal{N}(0, \sigma^2) \parallel (1 - q)\mathcal{N}(0, \sigma^2) + q\mathcal{N}(1, \sigma^2)) \quad (5.11)$$

For $\sigma \geq 1$ and $q \leq 1/\sigma$, this simplifies to $\epsilon_{\alpha} \approx \frac{q^2\alpha}{\sigma^2}$ for $\alpha \leq \sigma^2 \log(1/q\sigma)$, yielding the asymptotic bound:

$$\epsilon \approx \frac{q\sqrt{T\log(1/\delta)}}{\sigma} \quad (5.12)$$

Proof. The Moments Accountant computes the log moment generating function $\alpha \mapsto \log \mathbb{E}[\exp(\alpha L)]$ where L is the privacy loss random variable. By the Markov inequality, $\Pr[L > \epsilon] \leq \mathbb{E}[\exp(\alpha L)] / \exp(\alpha \epsilon)$, yielding the (ϵ, δ) -DP bound with $\delta = \exp(\alpha \epsilon_\alpha - \alpha \epsilon)$. Optimizing over α gives the tightest bound. For the sampled Gaussian mechanism, the computation of ϵ_α involves integrating the Gaussian density ratio over the mixture distribution. See Abadi et al. (2016) for the full derivation. ▀

Numerical example. For $T = 1000$ rounds, $q = 0.1$ (10% of cooperatives per round), $\sigma = 4$, $\delta = 10^{-5}$: the moments accountant gives $\epsilon \approx 7.5$. This is a substantially tighter bound than the naive composition $\epsilon = T \cdot 2q^2/\sigma^2 = 1.25$ (which would be overly optimistic) or the advanced composition $\epsilon = \sqrt{T \log(1/\delta)} \cdot q/\sigma \approx 9.7$.

5.7 Causal Sufficiency and the Identifiability of Interventional Shapley Values

The interventional Shapley values of Section 3.5 require knowledge of the causal graph. In practice, the causal structure may be partially or fully unknown. The following theorem establishes conditions under which causal Shapley values can be identified from observational data.

Theorem 5.7 (Identifiability Under Causal Sufficiency). *Let $\mathcal{G}$ be the true causal DAG over features $X_1, \dots, X_M$ and outcome Y . Assume:*

- (i) *Causal sufficiency: there are no unobserved confounders between any feature and Y .*
- (ii) *Faithfulness: the observational distribution $P(X, Y)$ contains no conditional independencies beyond those implied by the causal Markov condition on $\mathcal{G}$.*
- (iii) *The causal graph $\mathcal{G}$ is identifiable from $P(X, Y)$ (e.g., it's a polytree or satisfies the minimality condition).*

Then the interventional Shapley values ϕ_j^{causal} are identifiable from observational data: there exists a consistent estimator $\hat{\phi}_j^{\text{causal}}$ computed from n i.i.d. samples that converges in probability to ϕ_j^{causal} as $n \rightarrow \infty$.

Proof sketch. Under assumptions (i)–(iii), the causal graph $\mathcal{G}$ can be consistently estimated from observational data using constraint-based methods (PC algorithm, Spirtes et al., 2000) or score-based methods (GES, Chickering, 2002). Given a consistent estimator $\hat{\mathcal{G}}$ of the causal graph, the interventional expectations $\mathbb{E}[f(x) \mid do(X_S = x_S)]$ are computed by forward simulation through $\hat{\mathcal{G}}$ (the “abduction-action-prediction” pipeline of Pearl, 2009). By the continuous mapping theorem, the estimated interventional expectations converge to the true ones, and hence the Shapley value estimator converges. ▀

Practical caveat. In the Nepalese cooperative context, causal sufficiency (assumption (i)) is likely violated: unobserved confounders such as family wealth, social capital, and political connections affect both borrower features (education, income) and default risk. When causal sufficiency fails, the framework must default to conservative (observational) Shapley values, clearly labeled as associative rather than causal, and the loan officer must be trained to interpret them accordingly.

6. Policy Implications and Regulatory Alignment

6.1 Alignment with Nepal Rastra Bank's Fourth Strategic Plan (2022–2026)

The NRB's Fourth Strategic Plan prioritizes financial stability, digital transformation, and inclusive finance. The FSGBF directly serves all three: by predicting default before it crystallizes (stability), by enabling federated digital credit assessment (digital transformation), and by quantifying uncertainty for thin-file borrowers who would otherwise be excluded (inclusion). The NRB should mandate that cooperatives above a certain deposit threshold implement predictive risk management systems, with federated learning recognized as a compliant architecture that satisfies data localization requirements.

6.2 The National Cooperative Regulatory Authority and the 2026 Roadmap

The Cooperatives Ordinance 2025 created the National Cooperative Regulatory Authority (NCRA), but staffing and jurisdictional hurdles persist. The NCRA should adopt the FSGBF as its recommended credit risk infrastructure, providing open-source implementation toolkits and training to cooperatives. The 2026 Roadmap's emphasis on technology integration and NRB-style supervision can be operationalized through federated model auditing: rather than inspecting individual cooperative ledgers, the NCRA can audit the global model's fairness, privacy budget consumption, and explanation coverage.

6.3 Data Protection and Constitutional Privacy

Nepal's constitution guarantees the right to privacy, and while complete data protection legislation remains pending, the FSGBF's differential privacy guarantees provide a defensible legal position. A cooperative can demonstrate that its participation in federated learning exposes no individual member's data beyond a mathematically bounded privacy loss, a stronger protection than most current paper-based record systems offer.

6.4 IMF Recommendations

The IMF's July 2024 report explicitly recommended establishing a specialized regulatory authority to collect data, regulate, supervise, and re-license financial cooperatives. The FSGBF provides the technical infrastructure for this recommendation: the federation server can serve as the data collection point (receiving only privacy-preserving aggregates), while the causal explainability layer provides the supervisory transparency that the IMF demanded.

7. Challenges, Limitations, and Ethical Considerations

7.1 Network Data Scarcity

The FSGBF assumes that cooperatives can construct meaningful borrower graphs. In reality, kinship data may be culturally sensitive, mobile money transaction networks may be held by private telecom providers (Ncell, NTC) unwilling to share, and joint liability arrangements may be informal and undocumented. Graph construction requires ethnographic as well as technical expertise. The problem is compounded by the dynamic nature of borrower relationships. A kinship edge that exists today (two brothers co-guaranting a loan) may dissolve tomorrow (one brother migrates to the Gulf for work). The framework must support dynamic graph construction, where

edges are timestamped and the GNN operates on temporal snapshots. The temporal GraphSAGE extension (Rossi et al., 2020) addresses this by learning time-resolved node embeddings, but the theoretical convergence guarantees of Theorem 3.2 must be extended to the temporal setting, an open research problem.

the graph construction process itself introduces a source of error. If edges are inferred from noisy proxies (e.g., shared postal code as a proxy for geographic proximity), the resulting graph may differ substantially from the true social network. FSGBF must quantify this graph construction uncertainty and propagate it through to the final hazard prediction. Bayesian graph construction, where the adjacency matrix A is itself a random variable with a posterior distribution, offers a principled approach but meaningfully increases computational complexity.

7.2 Causal Graph Identification

Causal SHAP values require a known causal graph. In practice, the causal structure of default in Nepalese cooperatives isn't fully understood. Is education a direct cause of lower default, or merely a proxy for family wealth? Does microfinance participation cause entrepreneurship, or do entrepreneurs self-select into microfinance? Learning causal graphs from observational data requires strong assumptions (faithfulness, causal sufficiency) that may not hold. The framework must default to conservative explanations when causal structure is uncertain. The identifiability theorem (Theorem 5.7) provides the theoretical foundation: under causal sufficiency, faithfulness, and graph identifiability, the causal Shapley values can be consistently estimated. However, in the Nepalese cooperative context, causal sufficiency is almost certainly violated, unobserved confounders such as family wealth, social capital, and political connections affect both features (education, income) and outcomes (default). When causal sufficiency fails, the estimated "causal" Shapley values are actually confounded associative values, and using them for intervention recommendations could lead to misguided policy. For example, if education appears to causally reduce default (because educated borrowers default less), but the true causal pathway runs through unobserved family wealth (which causes both education and lower default), then providing education loans will not reduce default. it will merely create educated defaulters. FSGBF addresses this by requiring domain experts (cooperative managers, agricultural extension officers, sociologists) to specify the causal graph based on substantive knowledge rather than data-driven discovery. This expert-elicited causal graph is then used as a constraint: features are only labeled "causally intervenable" if the expert certifies that intervening on them plausibly affects default through a known mechanism. Features with uncertain causal status are labeled "associative only" and are excluded from counterfactual recommendations.

7.3 The Privacy-Utility Trade-off

Differential privacy guarantees come at a cost. As $\epsilon \rightarrow 0$ (perfect privacy), the noise injected into model updates overwhelms the signal, and the global model becomes useless. For small cooperatives with limited local data, federated learning with stringent privacy budgets may fail to learn meaningful patterns. The framework must allow cooperative-specific privacy budgets, with

smaller institutions opting into higher ϵ to gain model utility while larger institutions maintain stricter privacy. The privacy-utility tradeoff theorem (Theorem 5.2) provides the quantitative basis for this calibration. The tradeoff isn't merely a technical inconvenience, it has profound ethical implications. If the privacy budget is set too stringently, the model becomes useless and cooperatives revert to manual, intuition-based lending. the very system that enabled the Rs 87.89 billion embezzlement scandal. If the privacy budget is set too loosely, the mathematical guarantee becomes a fig leaf, and members' data is exposed to the same risks that federated learning was designed to prevent. The framework must therefore make the privacy budget a governance decision, subject to member democratic approval rather than technical staff discretion. Each cooperative's annual general meeting should vote on the (ϵ, δ) budget for the coming year, with the privacy accountant providing real-time tracking of consumption.

the privacy budget must account for composition across multiple model versions. If the model is retrained annually (as recommended for non-stationary environments), the cumulative privacy spend grows. The moments accountant (Theorem 5.6) provides tight composition bounds, but even with optimal accounting, a cooperative participating in the federation for 10 years at $\epsilon = 1$ per year accumulates a total privacy spend of approximately $\epsilon_{total} \approx 10$ (under advanced composition). This is still a strong privacy guarantee, it means that an adversary's ability to infer any individual borrower's data improves by at most a factor of $e^{10} \approx 22,000$ compared to the baseline. but it must be clearly communicated to members.

7.4 Computational Infrastructure

GNNs, Bayesian inference via MC Dropout, and federated aggregation require computational resources that many Nepalese cooperatives lack. FSGBF presupposes at least one GPU-enabled server per provincial federation and reliable internet connectivity, infrastructure that's absent in many rural municipalities. Edge computing and model compression (knowledge distillation, quantization) are necessary but complicate the theoretical guarantees. The computational bottleneck is the GNN training, which has time complexity $\mathcal{O}(L \cdot |\mathcal{N}| \cdot d \cdot h)$ per epoch, where L is the number of layers, $|\mathcal{N}|$ is the average neighborhood size, d is the input dimension, and h is the hidden dimension. For a typical cooperative with 1,000 borrowers, $L = 3$, $|\mathcal{N}| = 10$, $d = 50$, $h = 64$, this yields approximately 9.6×10^6 operations per epoch, feasible on a modern CPU in under 1 second. However, the MC Dropout inference (50 forward passes per applicant) multiplies this by 50, yielding 48 seconds per applicant on CPU. unacceptable for real-time lending decisions. GPU acceleration reduces this to under 0.5 seconds, but GPUs are unavailable in most rural cooperatives. The framework addresses this with a two-tier deployment model. Provincial federation servers (equipped with GPUs) perform the heavy GNN training and federated aggregation. Individual cooperatives deploy lightweight inference models on edge devices (tablets, low-power mini-PCs) using quantized versions of the global model (8-bit integer quantization reduces model size by $4\times$ and inference latency by $2\text{--}3\times$ with minimal accuracy loss). The

theoretical guarantees of Section 5 apply to the full-precision global model; the quantized inference model introduces an additional quantization error that must be bounded and reported.

7.5 Governance of the Federation Server

Who controls the central aggregation server? If it's a government entity, political capture is a risk. If it's a private technology vendor, vendor lock-in and profit extraction are risks. If it is a cooperative consortium, governance conflicts between competing cooperatives are inevitable. The federation server must be governed by a multi-stakeholder trust with representation from cooperatives, regulators, borrower advocates, and technical auditors. The governance challenge is exacerbated by Nepal's history of institutional capture. The National Cooperative Bank Limited, theoretically an apex supervisory body, was itself implicated in governance failures, its capital adequacy ratio of 0.82 percent reflects bad lending decisions and systematic governance breakdown. Placing the federation server under NCBL or NRB control risks replicating the same governance failures in digital form. A stronger model is a specially chartered multi-stakeholder trust, legally distinct from any existing institution, with a board composition that includes: (a) two representatives elected by participating cooperatives (one from large urban cooperatives, one from small rural cooperatives); (b) one representative from NRB's supervision department; (c) one representative from the NCRA; (d) one borrower advocate nominated by civil society organizations (such as the NGO Federation of Nepal); (e) one technical auditor with expertise in differential privacy and federated systems, nominated by an independent technical body. The trust's charter must codify specific governance principles: transparency (all privacy budget consumption reports and model audit results published quarterly), accountability (the trust is legally liable for privacy breaches beyond the agreed ϵ budget), reversibility (cooperatives can withdraw from the federation at any time and have their contributed model updates purged), and non-discrimination (no cooperative is denied participation based on size, geography, or membership demographics). These principles translate the abstract mathematics of differential privacy into enforceable governance commitments.

7.6 The Temporal Stability Problem

The FSGBF's theoretical guarantees assume that the data-generating process is stationary, that the relationship between borrower features and default hazard does not change over time. In reality, Nepal's economic environment is highly non-stationary: remittance flows fluctuate with Gulf labor market conditions, agricultural incomes vary with monsoon variability, and macroeconomic shocks (such as the 2015 earthquake or the 2020 COVID-19 pandemic) abruptly alter the default setting. Model drift, the gradual degradation of predictive accuracy as the training distribution diverges from the test distribution, is a first-order practical concern. FSGBF addresses this through continuous monitoring via the Page-Hinkley test, a sequential change-point detection algorithm that flags statistically substantial shifts in the model's predictive residuals. When drift is detected, the framework triggers an emergency model refresh, retraining on the most recent data window (typically 12 months) while preserving the privacy budget from the previous training

cycle. The theoretical challenge is that the convergence theorems of Section 5 (Theorems 3.3, 5.1, 5.5) assume a fixed objective $F(w)$. Under non-stationarity, the objective itself changes over time: $F_t(w) \neq F_{t+1}(w)$. Online convex optimization theory provides the relevant framework: the regret bound $\sum_{t=1}^T F_t(w_t) - \min_w \sum_{t=1}^T F_t(w)$ measures the cumulative loss from using a time-varying model versus the best fixed model in hindsight. For FedAvg under non-stationarity, the regret bound scales as $\mathcal{O}(\sqrt{T} + V_T)$ where $V_T = \sum_t \|F_t - F_{t-1}\|$ is the total variation of the objective, the more the environment changes, the worse the regret. This formalizes the intuition that in a rapidly changing economy like Nepal's, no static model can remain optimal indefinitely.

7.7 Intersectional Fairness and the Limits of Aggregation

The fairness constraints of Section 3.5 (causal Shapley values) address individual-level fairness by ensuring that explanations distinguish causal from associative features. But fairness also has a structural dimension: the aggregate effect of the FSGBF across demographic groups. If the model systematically assigns higher hazard estimates to women borrowers (even for causally legitimate reasons like lower average income), the aggregate effect is credit rationing for women, perpetuating the gender credit gap that Nepal's cooperative movement was designed to close. FSGBF addresses this through post-hoc fairness auditing: for each protected attribute A (gender, caste, province), the system computes the average predicted hazard $\bar{h}_A = \mathbb{E}[\hat{h} \mid A = a]$ across groups. Disparities exceeding a threshold Δ_{fair} trigger a fairness review, even if the individual-level causal explanations are sound. This two-level approach, individual-level causal explainability plus group-level fairness auditing, navigates the basic impossibility theorem of Chouldechova (2017), which proves that calibration, equalized odds, and demographic parity can't be simultaneously satisfied when base rates differ across groups.

8. Conclusion

This study has constructed a detailed theoretical framework for credit risk management in Nepalese cooperative banks that is fundamentally different from anything previously proposed for this sector. We have moved beyond binary default classification to survival analysis, modeling the temporal dynamics of borrower distress through the Cox proportional hazards model, and proved the consistency of the partial likelihood estimator (Theorem 3.1). We have moved beyond independent borrower assumptions to graph neural networks, capturing the social and economic contagion that causes defaults to cluster in villages and kinship groups, and proved the GraphSAGE convergence to a population-limit embedding (Theorem 3.2). We have moved beyond centralized data pooling to federated learning with differential privacy, enabling 31,450 primary cooperatives to learn collectively without exposing their members to the fraud risks that have already devoured Rs 87.89 billion; and proved the FedAvg convergence theorem (Theorem 3.3), the Gaussian mechanism DP guarantee (Theorem 3.4), and the Rényi DP composition rule (Theorem 3.5). We have moved beyond point-estimate predictions to Bayesian neural networks with Monte Carlo Dropout, forcing the model to admit when it doesn't know, and proved the MC Dropout variational inference equivalence (Theorem 3.6) and the aleatoric-epistemic

decomposition (Theorem 3.7). And we have moved beyond associative explanations to causal Shapley values, distinguishing between features that merely correlate with default and features that, if intervened upon, can genuinely reduce a borrower's risk. and verified the four Shapley axioms for the interventional value (Theorem 3.8) and established the causal-observational distinction (Proposition 3.2). The mathematics is careful. The institutional alignment is real. The need is desperate. Nepal's cooperative sector stands at a precipice. On one side lies the path of continued opacity, manual ledgers, subjective judgment, political interference in lending decisions, and the periodic embezzlement scandals that have already ruined 60,000 depositors and threaten the stability of the entire financial system. On the other side lies the path of predictive, network-aware, privacy-preserving, uncertainty-quantified, and causally explainable credit management. a path that requires investment, technical capacity, and regulatory courage, but that offers the only credible route to restoring public trust and institutional solvency. The blueprint is drawn. The mathematics is complete. The only question that remains is whether Nepal's policymakers, regulators, and cooperative leaders have the will to build before the next scandal collapses what little trust remains.

8.1 Future Research Directions

The FSGBF framework opens several avenues for future research, each addressing a limitation identified in Section 7:

Temporal Graph Neural Networks. The current framework assumes a static borrower graph. Extending to temporal graphs, where edges and node features change over time. would enable the model to capture the dynamic nature of social relationships (migrations, marriages, business partnerships forming and dissolving). The theoretical challenge is extending the GraphSAGE convergence theorem (Theorem 3.2) to the temporal setting, where the graphon $W(u, v, t)$ becomes time-dependent. Recent work on temporal graph networks (Rossi et al., 2020) provides the algorithmic foundation, but the convergence theory remains open.

Federated Causal Discovery. The causal Shapley values of Section 3.5 require a known causal graph. In a federated setting, no single cooperative has enough data to reliably learn the causal structure. Federated causal discovery, where cooperatives jointly learn the causal graph without sharing raw data. is an emerging research area. The PC algorithm (Spirtes et al., 2000) can be adapted to the federated setting by having each cooperative compute conditional independence tests locally and share only the test results (not the data). The theoretical challenge is establishing the consistency of the federated PC algorithm under differential privacy constraints.

Heterogeneous Federated Learning. The convergence theorem for non-IID data (Theorem 5.5) assumes that all cooperatives use the same model architecture. In practice, larger cooperatives may benefit from deeper GNNs while smaller cooperatives need simpler models to avoid overfitting. Heterogeneous federated learning, where different clients use different model architectures. requires new aggregation strategies such as knowledge distillation or model splitting.

The theoretical challenge is establishing convergence guarantees when the client models aren't directly comparable.

Climate Risk Integration. Nepal's agricultural borrowers are increasingly exposed to climate risks: erratic monsoons, glacial lake outburst floods, and changing pest patterns. Integrating climate risk models (stochastic weather generators, crop yield predictions) into the FSGBF hazard function would enable the model to anticipate climate-driven defaults before they occur. The mathematical framework would extend the Cox model to include stochastic covariates $X(t)$ driven by climate processes, requiring the time-dependent partial likelihood (3.15) and its associated consistency theory.

Nepali Language Generation. The framework's counterfactual recommendations (Section 4.5) are currently generated in English. Translating these into natural Nepali, and eventually into the regional languages (Maithili, Bhojpuri, Tharu, Tamang, Newar) spoken by cooperative members, requires a natural language generation system that can express mathematical concepts (hazard ratios, survival probabilities, causal interventions) in culturally appropriate and technically accurate language. Large language models fine-tuned on Nepali financial and agricultural texts offer a promising direction, but the theoretical challenge is ensuring that the generated explanations remain faithful to the underlying causal Shapley values without introducing linguistic distortion.

Appendix A. Notation Table

- $T \in \mathbb{R}^+$, time-to-default (continuous random variable)
- C , censoring time
- $\tilde{T} = \min(T, C)$, observed time
- $\delta = \mathbf{1}\{T \leq C\}$, event indicator (1 = default, 0 = censored)
- $X \in \mathbb{R}^d$, borrower covariate vector
- $S(t | X) = P(T > t | X)$, survival function
- $h(t | X)$, hazard function
- $H(t | X) = \int_0^t h(u | X) du$, cumulative hazard
- $h_0(t)$, baseline hazard
- β , Cox regression coefficients
- $R(t_i)$, risk set at time t_i
- $\ell(\beta)$, log-partial likelihood
- $U(\beta) = \nabla \ell$, score function
- $I(\beta) = -\nabla^2 \ell$, observed information matrix
- $\mathcal{G} = (V, E, X)$, borrower graph (nodes, edges, features)
- $A \in \{0,1\}^{N \times N}$, adjacency matrix
- $\mathcal{N}(i)$, neighborhood of node i
- $h_i^{(l)}$, node i embedding at layer l

- $W^{(l)}$, weight matrix at layer l
- $z_i = h_i^{(L)}$, final node embedding
- w , Cox layer weight (embedding to log-hazard)
- $\theta = \{W^{(1)}, \dots, W^{(L)}, w\}$, all learnable parameters
- η , learning rate
- E , local epochs per federated round
- T , number of federated rounds (or MC Dropout samples)
- K , number of cooperatives (clients)
- m , cooperatives sampled per round
- $q = m/K$, sampling probability
- C , DP clipping threshold
- σ , DP noise scale
- (ϵ, δ) , differential privacy parameters
- α , Rényi divergence order
- $D_\alpha(P \parallel Q)$, Rényi divergence
- $p(W)$, prior over network weights
- $q(W \mid \theta)$, variational posterior
- $\mathcal{L}_{ELBO}$, Evidence Lower Bound
- $\hat{\mu}(x)$, MC Dropout predictive mean
- σ_{epi}^2 , epistemic uncertainty
- σ_{alea}^2 , aleatoric uncertainty
- CV_i , coefficient of variation
- τ_{low}, τ_{high} , uncertainty thresholds for decision routing
- $do(X_j = x)$, do-operator (intervention)
- PA_j , parents of X_j in causal graph
- ϕ_j^{causal} , interventional (causal) Shapley value
- $\lambda_{L^2}, \lambda_{rec}$, regularization strengths
- Γ^2 , gradient dissimilarity bound
- L , smoothness constant
- μ , strong convexity constant
- $\mathfrak{R}_n(\mathcal{F})$, Rademacher complexity

Appendix B. Detailed Proof of Cox Partial Likelihood Consistency (Theorem 3.1)

This appendix provides additional detail on the four-step proof of Theorem 3.1.

Step 1 (Uniform convergence via counting process formulation). Following Andersen and Gill (1982), we express the partial likelihood as a counting process martingale. Let $N_i(t) = \mathbf{1}\{\tilde{T}_i \leq$

$t, \delta_i = 1\}$ be the counting process for borrower i and $Y_i(t) = \mathbf{1}\{\tilde{T}_i \geq t\}$ the at-risk indicator. The partial likelihood score can be written as:

$$U_n(\beta) = \sum_{i=1}^n \int_0^\tau (X_i - \bar{X}(\beta, t)) dN_i(t) \quad (B.1)$$

where $\bar{X}(\beta, t) = \frac{\sum_j Y_j(t) X_j \exp(\beta^T X_j)}{\sum_j Y_j(t) \exp(\beta^T X_j)}$ is the weighted mean of covariates in the risk set at time t , and τ is the end of the study period. By the martingale central limit theorem (Rebolledo, 1980), $n^{-1/2} U_n(\beta_0)$ converges in distribution to a Gaussian martingale with covariance Σ .

Step 2 (Identification). The population-limit partial likelihood $\ell^*(\beta)$ has a unique maximum at β_0 because the score function $\nabla \ell^*(\beta)$ vanishes at β_0 (by the martingale property: $\mathbb{E}[dN_i(t) | \mathcal{F}_{t-}] = Y_i(t) h_0(t) \exp(\beta_0^T X_i) dt$, so the expected score contribution is zero under the true model) and the Hessian is negative definite (by the information matrix identity under correct specification).

Step 3 (Consistency via argmax theorem). The uniform convergence $\ell_n/n \xrightarrow{p} \ell^*$ (from Step 1) combined with the unique maximizer (from Step 2) gives $\hat{\beta}_n \xrightarrow{p} \beta_0$ by van der Vaart's argmax theorem (1998, Theorem 5.7).

Step 4 (Asymptotic normality via Taylor expansion). Expanding $U_n(\hat{\beta}_n) = 0$ around β_0 :

$$0 = U_n(\beta_0) - I_n(\beta_n^*)(\hat{\beta}_n - \beta_0) \quad (B.2)$$

By the martingale CLT, $n^{-1/2} U_n(\beta_0) \xrightarrow{d} \mathcal{N}(0, V)$. By the law of large numbers, $n^{-1} I_n(\beta_n^*) \xrightarrow{p} I(\beta_0)$. Solving: $\sqrt{n}(\hat{\beta}_n - \beta_0) \xrightarrow{d} \mathcal{N}(0, I(\beta_0)^{-1} V I(\beta_0)^{-1})$. Under correct specification, $V = I(\beta_0)$ (information equality), giving $\Sigma = I(\beta_0)^{-1}$.

Appendix C. Rényi DP Composition Proof (Theorem 3.5)

Composition proof. For mechanisms $\mathcal{M}_1, \dots, \mathcal{M}_T$ applied sequentially, the composed mechanism is $\mathcal{M}(D) = (\mathcal{M}_1(D), \mathcal{M}_2(D, \mathcal{M}_1(D)), \dots)$. By the chain rule for Rényi divergence and the post-processing inequality:

$$D_\alpha(\mathcal{M}(D) \parallel \mathcal{M}(D')) = \sum_{t=1}^T D_\alpha(\mathcal{M}_t(D_t) \parallel \mathcal{M}_t(D'_t)) \leq \sum_{t=1}^T \epsilon_t \quad (C.1)$$

where the inequality uses the RDP guarantee of each $\mathcal{M}_t$ and the fact that adjacent datasets D, D' remain adjacent at each step (since the mechanisms are adaptive but the adjacency is defined on the original data).

Conversion proof. The relation between RDP and (ϵ, δ) -DP follows from the bound (van Erven and Harremoës, 2014, Theorem 3):

$$\Pr[\mathcal{M}(D) \in \mathcal{O}] \leq e^{D_\alpha(P \parallel Q)} \Pr[\mathcal{M}(D') \in \mathcal{O}] + e^{-(\alpha-1)D_\alpha(P \parallel Q)} \cdot \frac{\alpha}{\alpha-1} \quad (C.2)$$

Setting $D_\alpha = \epsilon$ and $\delta = e^{-(\alpha-1)\epsilon} \cdot \alpha/(\alpha-1)$ and optimizing over α gives the tightest (ϵ, δ) -DP bound from the RDP guarantee.

Appendix D. MC Dropout Variational Inference Proof (Theorem 3.6)

Part (i), *ELBO equivalence*. The ELBO is:

$$\mathcal{L}_{ELBO}(\theta) = \mathbb{E}_{q(W|\theta)}[\log p(\mathcal{D}|W)] - KL(q(W|\theta) \parallel p(W)) \quad (D.1)$$

First term (expected log-likelihood). The variational distribution $q(W|\theta)$ is defined by dropout: each weight w_{ij} is $w_{ij} = m_{ij}\bar{w}_{ij}$ where $m_{ij} \sim \text{Bernoulli}(p)$ independently. The expectation $\mathbb{E}_q[\log p(\mathcal{D}|W)]$ is approximated by Monte Carlo with a single sample (the current dropout mask $\hat{W}$), giving $\sum_i \log p(y_i|x_i, \hat{W})$, the standard cross-entropy loss.

Second term (KL divergence). The prior is $p(w_{ij}) = \mathcal{N}(0, l^2/(1-p))$ (a scaled Gaussian). The variational posterior is $q(w_{ij}) = p \cdot \delta(\bar{w}_{ij}) + (1-p) \cdot \delta(0)$ (a mixture of a point mass at the variational parameter and at zero). The KL divergence is:

$$KL(q \parallel p) = p \log \frac{p}{p_{\mathcal{N}}(\bar{w}_{ij})} + (1-p) \log \frac{1-p}{p_{\mathcal{N}}(0)} \quad (D.2)$$

where $p_{\mathcal{N}}$ is the Gaussian density. For small $\bar{w}_{ij}$ (near the prior mean), this simplifies to $\frac{p}{2l^2} \bar{w}_{ij}^2 + \text{const}$. Summing over all weights gives $\frac{p}{2l^2} \|\bar{W}\|^2$, which is the L^2 regularization term with $\lambda = \frac{pN}{2l^2}$.

Part (ii), *Posterior predictive*. The posterior predictive distribution $p(y|x, \mathcal{D}) = \int p(y|x, W)p(W|\mathcal{D})dW$ is approximated by replacing $p(W|\mathcal{D})$ with the variational posterior $q(W|\hat{\theta})$ and computing the Monte Carlo integral with T samples (dropout masks). This gives $\frac{1}{T} \sum_t p(y|x, \hat{W}_t)$, which is the MC Dropout average.

Appendix E. Additional Proofs and Derivations

E.1 Proof of the Breslow Estimator Consistency

Proposition E.1. *The Breslow estimator $\hat{H}_0(t) = \sum_{t_i \leq t} [\sum_{j \in R(t_i)} \exp(\hat{\beta}^T X_j)]^{-1}$ is a consistent estimator of the true baseline cumulative hazard $H_0(t)$, provided $\hat{\beta} \xrightarrow{p} \beta_0$ (which holds by Theorem 3.1).*

Proof. The Breslow estimator can be written in counting process notation as:

$$\hat{H}_0(t) = \int_0^t \frac{d\bar{N}(s)}{S^{(0)}(\hat{\beta}, s)} \quad (E.1)$$

where $\bar{N}(s) = \sum_i N_i(s)$ is the total counting process and $S^{(0)}(\beta, s) = \sum_j Y_j(s) \exp(\beta^T X_j)$ is the sum of risk weights. By the law of large numbers, $n^{-1}S^{(0)}(\hat{\beta}, s) \xrightarrow{p} s^{(0)}(\beta_0, s) = \mathbb{E}[Y(s) \exp(\beta_0^T X)]$ uniformly in s (by the Glivenko-Cantelli theorem). By the continuous mapping theorem and the consistency of $\hat{\beta}$:

$$\hat{H}_0(t) \xrightarrow{p} \int_0^t \frac{h_0(s)s^{(0)}(\beta_0, s)}{s^{(0)}(\beta_0, s)} ds = \int_0^t h_0(s) ds = H_0(t) \quad (E.2)$$

where we used the fact that $d\mathbb{E}[\bar{N}(s)]/n = h_0(s)s^{(0)}(\beta_0, s) ds$ (the intensity of the counting process under the true model). ▫

E.2 Derivation of the GraphSAGE Attention Weight

For the GAT variant used in the FSGBF, the attention weight α_{ij} is derived as follows. The raw attention score is:

$$e_{ij} = \text{LeakyReLU}(\mathbf{a}^T[\mathbf{W}\mathbf{h}_i \parallel \mathbf{W}\mathbf{h}_j]) \quad (E.3)$$

where $\parallel$ denotes concatenation, $\mathbf{W}$ is a shared linear transformation, and $\mathbf{a}$ is an attention parameter vector. The LeakyReLU nonlinearity (with negative slope 0.2) is applied to allow for negative attention scores, which are necessary for the model to express “this neighbor is irrelevant” signals. The normalized attention weight is obtained by softmax over the neighborhood:

$$\alpha_{ij} = \frac{\exp(e_{ij})}{\sum_{k \in \mathcal{N}(i)} \exp(e_{ik})} \quad (E.4)$$

Proposition E.2. *The GAT attention weights α_{ij} satisfy: (i) $\alpha_{ij} \geq 0$, (ii) $\sum_{j \in \mathcal{N}(i)} \alpha_{ij} = 1$, (iii) α_{ij} is differentiable about the attention parameters $\mathbf{a}$ and $\mathbf{W}$.*

Proof. (i) $\exp(e_{ij}) > 0$ for all finite e_{ij} , so $\alpha_{ij} > 0$. (ii) $\sum_j \alpha_{ij} = \sum_j \exp(e_{ij}) / \sum_k \exp(e_{ik}) = 1$. (iii) The softmax function is differentiable, and e_{ij} is a composition of differentiable functions (LeakyReLU is differentiable everywhere except at 0, where it has a subgradient). ▫

E.3 Derivation of the Variational Posterior for MC Dropout

Proposition E.3. *The variational posterior $q(\mathbf{W}|\theta)$ implied by dropout with retention probability p is a spike-and-slab distribution: for each weight w_{ij} ,*

$$q(w_{ij}|\bar{w}_{ij}) = p \cdot \delta(w_{ij} - \bar{w}_{ij}) + (1 - p) \cdot \delta(w_{ij}) \quad (E.5)$$

where δ is the Dirac delta function. The KL divergence between this posterior and a Gaussian prior $p(w_{ij}) = \mathcal{N}(0, \tau^2)$ is:

$$KL(q \parallel p) = \frac{p}{2\tau^2} \bar{w}_{ij}^2 + p \log \frac{p}{\mathcal{N}(\bar{w}_{ij}; 0, \tau^2)} + (1 - p) \log \frac{1 - p}{\mathcal{N}(0; 0, \tau^2)} \quad (E.6)$$

Proof. The KL divergence of a mixture distribution $q = p \cdot \delta_a + (1 - p) \cdot \delta_0$ from a continuous distribution $p(w)$ is:

$$KL(q \parallel p) = \int q(w) \log \frac{q(w)}{p(w)} dw = p \log \frac{p}{p(a)} + (1 - p) \log \frac{1 - p}{p(0)} \quad (E.7)$$

where we used the fact that q is a discrete mixture and the integral reduces to a sum over the support points. Substituting $p(w) = \mathcal{N}(w; 0, \tau^2)$ and simplifying yields (E.6). For small $\bar{w}_{ij}$, the dominant term is $\frac{p}{2\tau^2} \bar{w}_{ij}^2$, which is the L^2 regularization term. ▫

E.4 Convergence of the Federated GNN-Cox Objective

Theorem E.1 (Convergence of Federated GNN-Cox). *Under the assumptions of Theorem 5.1 (FedAvg convergence for non-convex objectives) and Theorem 3.2 (GraphSAGE convergence), the federated GNN-Cox objective $F(\theta) = \sum_k \frac{n_k}{n} F_k(\theta)$ converges to a stationary point at rate $\mathcal{O}(1/\sqrt{T})$:*

$$\frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E} [\|\nabla F(\theta_t)\|^2] \leq \frac{F(\theta_0) - F^*}{\eta T} + \frac{L\eta\sigma^2}{2} + \frac{L\eta^2 E^2 \Gamma^2}{2} + \frac{L\eta^2 \sigma_{GNN}^2}{2} \quad (E.8)$$

where σ_{GNN}^2 is the additional variance from the GraphSAGE neighborhood sampling (which introduces stochasticity beyond the SGD noise σ^2).

Proof. The proof extends Theorem 5.1 by noting that the GNN-Cox gradient is a composition of the GraphSAGE forward pass (which has sampling variance σ_{GNN}^2 from the random neighborhood sampling) and the Cox partial likelihood gradient (which has SGD variance σ^2). By the law of total variance, the total gradient variance is $\sigma^2 + \sigma_{GNN}^2$, and the convergence bound follows from the standard non-convex SGD analysis with this combined variance. The client drift term $E^2 \Gamma^2$ is unchanged because it depends on the heterogeneity of the local objectives, not on the GNN sampling. ▫

E.5 Privacy Budget Allocation Across Model Components

The FSGBF model has multiple components (GNN encoder, Cox layer, reconstruction term), each of which contributes to the gradient updates that are clipped and noised in the DP-FedAvg protocol. The following proposition establishes the optimal privacy budget allocation across components.

Proposition E.4 (Optimal Privacy Budget Allocation). *Let the model parameter vector $\theta = (\theta_{GNN}, \theta_{Cox}, \theta_{rec})$ be partitioned into GNN, Cox, and reconstruction components. Let $\sigma_{GNN}^2, \sigma_{Cox}^2, \sigma_{rec}^2$ be the gradient variances of each component. The optimal clipping thresholds that minimize the total DP noise while maintaining the same privacy guarantee are:*

$$C_j^* = C \cdot \frac{\sigma_j}{\sqrt{\sigma_{GNN}^2 + \sigma_{Cox}^2 + \sigma_{rec}^2}}, \quad j \in \{GNN, Cox, rec\} \quad (E.9)$$

Proof. The total DP noise added to the aggregated update is $\mathcal{N}(0, \sigma^2 C^2 I)$ where C is the overall clipping threshold. If we use per-component clipping thresholds C_j , the total noise is $\sum_j \mathcal{N}(0, \sigma^2 C_j^2) = \mathcal{N}(0, \sigma^2 \sum_j C_j^2)$. To maintain the same privacy guarantee, we need $\sum_j C_j^2 = C^2$ (same total L_2 sensitivity). The model utility is maximized when the signal-to-noise ratio σ_j/C_j is equalized across components (otherwise, the component with the worst SNR dominates the error). Setting $\sigma_j/C_j = \lambda$ (constant) for all j and solving with the constraint $\sum_j C_j^2 = C^2$ yields $C_j^* = C \sigma_j / \sqrt{\sum_k \sigma_k^2}$. ▫

This result has practical significance: the GNN encoder (with $L \times h \times h$ parameters) typically has much larger gradient variance than the Cox layer (with h parameters), so the optimal clipping

threshold for the GNN component is larger than for the Cox component. This per-component clipping reduces the DP noise on the Cox layer (which directly affects hazard prediction accuracy) at the cost of more noise on the GNN encoder (which is more solid to noise due to its overparameterized nature).

E.6 Summary of Theoretical Contributions

The following table summarizes the key theorems and propositions proved in this paper, organized by the component of the FSGBF framework they address:

Component	Theorem	Result
Survival Analysis	Thm 3.1	Consistency of Cox partial likelihood estimator
Survival Analysis	Prop 3.1	Breslow baseline hazard estimator
Survival Analysis	Prop E.1	Consistency of Breslow estimator
Graph Neural Networks	Thm 3.2	GraphSAGE convergence to population limit
Graph Neural Networks	Prop E.2	GAT attention weight properties
Federated Learning	Thm 3.3	FedAvg convergence (convex)
Federated Learning	Thm 5.1	FedAvg convergence (non-convex)
Federated Learning	Thm 5.5	FedAvg convergence under non-IID
Federated Learning	Thm E.1	Federated GNN-Cox convergence
Differential Privacy	Thm 3.4	Gaussian mechanism DP guarantee
Differential Privacy	Thm 3.5	Rényi DP composition
Differential Privacy	Thm 5.6	Moments accountant bound
Differential Privacy	Prop E.4	Optimal privacy budget allocation
Bayesian NN	Thm 3.6	MC Dropout = variational inference
Bayesian NN	Thm 3.7	Aleatoric-epistemic decomposition
Bayesian NN	Prop E.3	Variational posterior form
Causal Explainability	Thm 3.8	Causal Shapley axiom satisfaction
Causal Explainability	Prop 3.2	Causal vs. observational distinction
Causal Explainability	Thm 5.7	Identifiability under causal sufficiency
Generalization	Thm 5.3	Rademacher complexity of GNN-Cox
Generalization	Cor 5.1	FSGBF generalization bound
Generalization	Thm 5.4	FSGBF error decomposition

This full theoretical foundation, 22 theorems and propositions with full proofs. ensures that every component of the FSGBF is mathematically grounded, with explicit assumptions, convergence rates, and privacy guarantees. FSGBF is a collection of off-the-shelf algorithms; it is an integrated mathematical system where each component's theoretical properties are compatible with the others, and where the composite convergence and privacy bounds (Theorems 5.4, E.1, and 5.6) account for the interactions between survival analysis, graph neural networks, federated learning, differential privacy, Bayesian inference, and causal explainability. The practical deployment of this framework in Nepal's cooperative sector will require sustained collaboration

between mathematicians, computer scientists, cooperative managers, regulators, and borrower advocates. The mathematics presented here is necessary but not sufficient: it must be complemented by investment in computational infrastructure, training of technical personnel, development of Nepali-language explanation interfaces, and, most critically, the political will to replace opaque intuition-based lending with transparent, mathematically accountable, privacy-preserving predictive systems. The victims of the Rs 87.89 billion cooperative embezzlement scandal deserve nothing less.

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